

Climate Change and Risk Management: Existence Conditions for Efficiency Optimising Insurance Portfolios with Low-probability High-impact Risks

Borbála Szüle^{1*}

¹ Institute of Operations and Decision Sciences, Corvinus University of Budapest, Fővám tér 8., H-1093 Budapest, Hungary

* Corresponding author, e-mail: borbala.szule@uni-corvinus.hu

Received: 05 July 2023, Accepted: 31 October 2025, Published online: 18 December 2025

Abstract

Climate change may be associated with an increasing frequency of natural catastrophes, and possible ways of alleviating at least the related financial burden are of high importance. Catastrophe probabilities are low, but catastrophic events can have overwhelming effects, and insurance market solutions for catastrophe risk management are often unavailable. By focusing on the insurance industry's perspective, in which risk and profitability can be key decision factors, this paper aims to highlight a possible theoretical obstacle impeding further market development. In a stylised model the findings suggest that an essential feature of catastrophe risks (the inverse relationship between risk probability and risk impact) may lead to a situation where the theoretical availability of efficient insurance for low-probability high-impact risk is highly limited. This result underlines the value of appropriate catastrophe insurance and reinsurance product design that could theoretically contribute to alleviating efficiency problems.

Keywords

climate change, insurance, insurance companies, risk assessment, probability

1 Introduction

Catastrophes have long accompanied human society, and with the ongoing climate change, their effects can be even more devastating. Catastrophe insurance could mitigate at least the financial losses, but it is not a uniquely prevalent risk management solution worldwide. It is therefore vitally important to understand why catastrophe insurance is so scarce. Theoretical premises for the existence of insurance include the idea that insurance optimises utility for both buyers and insurers. This paper examines the perspective of insurers.

Insurance policies are sometimes available for low-probability high-impact risks (Charpentier and Le Maux, 2014; Zhao et al., 2020), but some risks with low probability and high severity remain uninsured (Louaas and Picard, 2018). Global losses from natural catastrophes amounted to approximately 280 billion dollars in 2021, and only around 42 percent of it was insured (Münchener Rückversicherungs-Gesellschaft, NatCatSERVICE, 2022).

Modelling and even defining catastrophe risk is not straightforward. A broad definition of a catastrophic event could focus on the usually small probability of realisation

that is accompanied by an extremely large loss (Grechuk and Zabarankin, 2014). The modelling of these low-probability high-impact risks usually requires several theoretical elements (Embrechts et al., 1997), since relevant historical data is quite rare (Wirtz et al., 2014).

This paper aims to contribute to the literature with a theoretical examination of reasons for the scarcity of insurance for low-probability high-impact risks. Previous literature has studied this topic extensively, and numerous aspects have been highlighted. Catastrophic risk can increase insurance premium (Duncan and Myers, 2000), and for some loss distributions insurers with solvency constraints may even charge a price that is many times the expected loss, meaning that it may be rational to forego disaster insurance (Kousky and Cooke, 2012). Some individuals may perceive catastrophe probabilities and the present value of future benefits as relatively low, which can also contribute to a low investment in loss reduction measures (Kunreuther, 1996), or individuals can face costs to discovering the true probability of a rare event that may impede insurance purchase (Kunreuther and Pauly, 2004).

Catastrophe insurance related financial losses may drain the capital capacity of insurers (Hofer et al., 2020), but, with adequate risk management, the impact on insurers is not necessarily severe (Hagendorff et al., 2015). Catastrophe risk can be diversified (Wu, 2020), and other external risk transfer mechanisms such as reinsurance and securitisation can also contribute to manage catastrophe risks (Burnecki et al., 2019; Hofer et al., 2020; Subramanian and Wang, 2018). Besides the widespread use of insurance-linked securities, for example catastrophe bonds (Burnecki et al., 2019), reinsurance is a dominant risk transfer mechanism for catastrophe risk (Subramanian and Wang, 2018). Nevertheless, Ibragimov et al. (2009) identify a "non-diversification trap" by showing that it may be optimal not to offer insurance for catastrophic risk and not to participate in reinsurance markets when dealing with risk distributions that have a heavy left tail. Risk diversification and securitisation can also be complemented by government interventions (Wu, 2020).

According to Charpentier and Le Maux (2014), if the insurer has a non-zero probability of insolvency, government-provided insurance can be more attractive in terms of expected utility for some types of natural catastrophe risks. Goodspeed and Haughwout (2012) demonstrate that federal disaster insurance may create an externality and result in a relatively low pre-disaster protective investment if regional governments do not cooperate when undertaking disaster prevention measures. Wu (2020) points out that in the presence of an increasing probability of catastrophe loss government disaster bailouts may reduce the demand for insurance.

The paper aims to contribute to literature by focusing on another aspect when examining catastrophe insurance prevalence. From an insurer perspective, by explicitly modelling the inverse relationship between the loss size and occurrence probability (that characterises low-probability high-impact risks) an efficiency measure is defined and it is examined, whether an optimal (maximum) efficiency level exists, if the insurer may choose the insurance risk level. The everyday activities of insurers are of course far more complex than what is modelled by the presented theoretical framework, but the main focus of the analysis is on the effect of the inverse relationship between the insurance risk and the loss size, and this may be highlighted by the model. The efficiency measure is based on the profitability and institution-level risk (insolvency risk), and it is examined whether an insurance with low probability and high possible loss size may be optimal from the point of view of efficiency.

For the sake of simplicity, the applied modelling approach focuses on the comparison of risk and profitability, and insurance portfolio composition effects are not taken into account, although in practice these effects may be important. For instance, Wu (2015) points out that issuing life insurance may reduce the impact of a large catastrophe claim on financial stability. In the presented model insurance risk is assumed to be homogeneous.

It is important to emphasise that there is a difference between the policy-level insurance risk (the probability of insurance event) and the institution-level risk (measured by the insolvency probability). Similar risk measures for institution-level risk have been analysed in previous literature, for instance, Luo et al. (2008) examines the probability of ruin, in a risk related context Bayraktar et al. (2009) indicates standard deviation per policy, while Cummins et al. (2002) analyses the standard deviation of average losses per policy.

Efficiency is examined to capture both profitability and institution-level risk effects, its measure is the ratio between contract-level profitability and insolvency probability. The paper aims to contribute to previous literature by examining the question, whether an optimal insurance risk level exists that maximises this measure, and whether (and under which conditions) this possible optimum belongs to a low-probability-high-impact insurance risk. These results may contribute to explaining the gap between insured and total losses belonging to natural catastrophes from an insurer's point of view.

The paper is organised as follows. Section 2 introduces the theoretical model of the insurer. Section 3 presents results about the relationship between insurance risk level and components of the efficiency measure, while Section 4 highlights the results about efficiency. The conclusions of the paper are summarised in Section 5.

2 The model

Insurance business is complex, and the stylised model presented in this paper incorporates only selected essential features of "traditional" (life or non-life) insurance activity. One of these features is that on the asset side of insurer balance sheets the majority of assets is related to bonds, and the largest component on the other side of the balance sheet represents insurance liabilities (Insurance Europe, 2014:p.23). The model reflects these empirical findings. For the sake of simplicity, homogeneity of insurance policies is assumed: in the model the insurance event occurrence probability (indicated by p) and the sum insured (indicated by B) is the same for all

insurance policies. The size of the insurance portfolio is measured by the number of insurance policies (indicated by n). Although heterogeneity of insurance policies may have significant institution-level risk effects, for example a natural hedging strategy may contribute to deal with longevity risk in insurance companies (Tsai et al., 2010; Wang et al., 2010), and the product mix can also lower the shortfall risk of an insurer (Bohnert et al., 2015), the paper focuses on the effects that may be induced by the relationship between the probability and potential impact of insurance events.

Therefore, in the model a key assumption is related to the characterisation of low-probability high-impact insurance risks. According to the assumptions, the sum insured (B) depends on the probability of the insurance event occurrence (p) so that $\partial B(p)/\partial p < 0$, a higher probability is associated by a lower sum insured. It is also assumed that $(\partial^2 B(p))/(\partial p^2) < 0$. These assumptions are primarily of theoretical importance and reflect the difference that may exist between catastrophe insurance and other types of insurance contracts. However, it is important to emphasise that this relationship between the sum insured (B) and the probability of insurance event occurrence (p) is not necessarily observable for all possible sets of insurance policies.

Catastrophe risk may be considered a relatively new or at least changing type of risk in insurance that is usually associated with low probability events (Gibson et al., 2014). Events with low probability and high consequences include for example natural catastrophes such as earthquake (Goda and Hong, 2008), flood (Lamond and Penning-Rowsell, 2014); a financial crisis may also be an example (Robinson and Botzen, 2019). Catastrophic climate risks may also be associated with economic losses (Dietz, 2011). Compared to catastrophe insurance, other "traditional" types of insurance may be associated with higher event occurrence probability and lower possible financial consequences. In some insurance product design cases (for example in health insurance) an insurer could consider whether it should focus on offering coverage of high-probability/low-cost events or offering coverage of relatively expensive events that are associated with a very low probability (Schram and Sonnemans, 2011). In the model, the assumed inverse relationship between the sum insured (B) and the insurance event probability (p) aims to highlight these differences.

For each individual insurance policy a random variable (indicated by $\xi_j, j = 1, \dots, n$) can be defined, so that the value of this random variable is equal to 1 if the insurance event occurs in case of the j -th insurance policy and 0 otherwise.

The sum of these random variables is the total number of occurred insurance events (indicated by ξ). It is assumed that the individual insurance events (indicated by $\xi_j, j = 1, \dots, n$) are independent. The total number of occurred insurance events is assumed to follow normal distribution, since theoretically its binomial distribution function can be approximated with the normal distribution function for a sufficiently large number of insurance policies.

In the model, it is assumed that policyholders pay a single premium (net premium plus certain expenses), and that policyholders receive the sum insured (B) if the insurance event occurs during the term of the insurance. It is also assumed that the insurance term is one year. Without loss of generality, it can be assumed that the model also includes the technical rate of return (indicated by i) for the calculation of the insurance premium.

Based on the model assumptions, and by applying the equivalence principle (Dickson et al., 2011:p.146), the net premium payable by the policyholder at the beginning of the insurance term is $(B(p) \cdot p)/(1 + i)$. At the beginning of the insurance term the sum of collected net premiums is equal to the value of the insurance reserves (that corresponds to insurance liabilities). The value of own funds (equity) is assumed to be related to the value of insurance liabilities, so that the equity value is equal to $(B \cdot p \cdot n \cdot s)/(1 + i)$, where s indicates a "solvency multiplier". It is also assumed that the own funds of the insurance company in the model are higher than the regulatory requirement.

In the model, it is assumed that the collected premiums and the own funds are invested so that investments correspond to regulatory requirements and company-level asset-liability management decisions as well. The value of the return on these investments (that part of the return that belongs to the insurance company, according to regulations) is indicated by r .

3 Profitability and risk measures

The one-year loss (and its negative, the profit value) is a random variable, represents riskiness, and it can be applied to define several risk measures. Efficiency of insurance is driven by both profitability and risk. Theoretically efficiency measures could be defined with an arbitrary selection of weighting parameters for profitability and risk measures. In this paper, the efficiency measure is defined as the ratio between one of the possible profitability measures (the contract-level expected value of profit) and the insolvency probability of the insurer. The key question in the analysis is whether there is an insurance risk level (p) that maximises this efficiency measure.

Under the model assumptions profitability can be measured by the random variable representing one-year profit, as described by Eq. (1), at the end of the insurance term the profit is a random variable that follows normal distribution:

$$\mu = B(p) \cdot p \cdot \pi \cdot \frac{(1+s) \cdot (1+r)}{1+i} - B(p) \cdot \xi. \quad (1)$$

The expected value of the profit is $B(p) \cdot p \cdot \pi \cdot (((1+s) \cdot (1+r))/(1+i) - 1)$, and the standard deviation is $B(p) \cdot \sqrt{\pi \cdot p \cdot (1-p)}$. It is important to note that the insurance risk level (p) influences the profit. The expected value of the profit per policy is $m(p) = E(\mu)/n$, and in the model $m(p) = B(p) \cdot p \cdot (((1+s) \cdot (1+r))/(1+i) - 1)$. Technically, it is a policy-level measure, but it also captures institution-level profitability adequately.

In addition to profitability, another major question is how riskiness could be adequately quantified. Risk is a complex issue, and there are numerous ways to quantify it. Blanchet and Lam (2013) point out that there are several modelling approaches; in addition to the classical risk theory that assumes aggregated claim and premium processes, there is also a bottom-up approach that builds the risk process from micro-level parameters. The modelling approach adopted in this paper is similar, although not identical, to the basic principles of this bottom-up approach.

During risk measurement, the first step is often the estimation of the loss distribution, and then a risk measure is computed (e.g., Cont et al., 2010). Some risk measures are related to the quantiles, for example the expected shortfall value (Bignozzi et al., 2018). Indicators that are associated with the moments of a distribution are also often applied to measure risk, for example the standard deviation of the return (e.g., Markowitz, 1952). The moments of a distribution can be important in measuring risk (e.g., Fama and French, 2004; Lintner, 1965; Sharpe, 1964; Vendrame et al., 2016). In insurance, the moments belonging to the present value of benefit payments or loss random variables can be essential in risk measurement, for instance the mean and standard deviation of the prospective loss variables may be calculated (Chen et al., 2017). The standard deviation per policy can vanish if the number of claims is large and the claims are associated with independent random variables (Bayraktar et al., 2009).

Although theoretically a distinction between systematic and unsystematic risk can also be made so that "well-diversified" portfolios often can be assumed to exhibit no unsystematic risk, only systematic risk (e.g., Khan and Sun, 2003), this paper assumes that the number of

insurance policies is large enough for achieving results without considerable non-diversification problems so that the unsystematic risk may account only for a very modest part of the institution-level risk. It is also worth noting that in some cases, for example for extremely heavy-tailed risks with unbounded distribution support, diversification may increase risk (Ibragimov and Walden, 2007), but under the model assumptions, these theoretically possible diversification problems do not influence the results.

The theoretical properties of risk measures may also influence risk measurement. In some cases, coherent risk measures (Artzner et al., 1999) may be preferable, although a coherent risk measure does not necessarily fulfil all possible theoretical requirements. For instance, Cont et al. (2010) point out that a conflict may exist between the subadditivity (that is one of the properties of coherent risk measures) and robustness of risk measurement processes.

Theoretically, several risk measures could indicate institution-level risk, for example Cummins et al. (2002) calculate the standard deviation of average losses per policy, while Gibson et al. (2014) measure risk by the standard deviation of losses, and the probability of insolvency is also one of the possible risk measures.

According to model assumptions, the insolvency probability of the insurer at the end of the insurance term is described by Eq. (2), where the function $\Phi(x)$ is the distribution function of the standard normal distribution:

$$\Phi\left(-\frac{B(p) \cdot p \cdot \pi \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1\right)}{B(p) \cdot \sqrt{\pi \cdot p \cdot (1-p)}}\right). \quad (2)$$

The insolvency probability depends on the level of insurance risk level (indicated by p in the model), therefore theoretically it can be examined, whether an insolvency probability minimising insurance risk level exists. It can be concluded that the sign of the first derivative of:

$$B(p) \cdot p \cdot \pi \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1\right) / \left(B(p) \cdot \sqrt{\pi \cdot p \cdot (1-p)}\right),$$

with respect to p is positive for all p (for all insurance risk levels), and it can be interpreted so that if (assuming that no other parameters change) the insurance risk level (the value of p) increases, then the value of:

$$B(p) \cdot p \cdot \pi \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1\right) / \left(B(p) \cdot \sqrt{\pi \cdot p \cdot (1-p)}\right)$$

also increases, thus the insolvency probability decreases. These results suggest that a higher insurance event probability (p) is associated with a lower insolvency probability. Although this finding is quite straightforward, it is a key result when examining low-probability high-impact risks.

4 Insurance portfolio efficiency

Efficiency is often studied in an economic context. For instance, the exploration of efficient portfolio features is an essential ingredient of portfolio theory. It is possible that a positive relationship exists and higher return may be associated with higher risk (e.g., Lintner, 1965; Markowitz, 1952; Merton, 1973; Sharpe, 1964), although other papers (e.g., Backus and Gregory, 1993) argue that theoretically various relations between risk premiums and conditional variances can be found. This relationship may be affected by several factors, for example the set of influencing factors may include the size of the dataset (Lundblad, 2007), the application of the linear assumption in the risk-return relationship (e.g., Salvador et al., 2014), the market return values (e.g., Marks and Nam, 2018), the contemporaneous correlation between the return and realised variance (e.g., Yang, 2019), and the assumptions about the volatility feedback effect (e.g., Wang and Yang, 2013).

Risk and return (or profitability) are also important for insurers. These two aspects may be combined, for example the mean-variance utility function of the insurer could be maximised (e.g., Golubin, 2006). In this paper, the efficiency measure is defined as the ratio between a profitability and an institution-level risk measure. Equation (3) indicates the efficiency measure:

$$F(p) = \frac{B(p) \cdot p \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right)}{\Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right)} \quad (3)$$

This efficiency measure depends on the insurance risk level (p , indicating the probability of the insurance event occurrence). To model a basic feature of catastrophic events, a key assumption is that the relationship between the sum insured (B) and the probability of the insurance event occurrence is inverse: for example, in case of a low-probability high-impact event, a higher sum insured (B) is associated with a lower insurance event probability. In the stylised model the insurer may select the composition of the insurance portfolio, and it corresponds to selecting the insurance event risk. The main question is

whether (under certain conditions) the efficiency optimising insurance event risk may belong to a low-probability high-impact catastrophic event.

To explore this question, the mathematical condition:

$$\frac{\partial F(p)}{\partial p} = 0$$

is examined. Under the model assumptions:

$$\frac{\partial F(p)}{\partial p} = V_1 + V_2 + V_3,$$

where:

$$V_1 = \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot p \cdot \frac{\partial B(p)}{\partial p},$$

$$V_2 = \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot B(p),$$

$$V_3 = B(p) \cdot p \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right)^2 \cdot \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \frac{1}{2} \cdot \sqrt{\frac{1-p}{p}} \cdot \frac{1}{(1-p)^2} \cdot \sqrt{\pi}.$$

$$V_3 = B(p) \cdot p \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right)^2 \cdot \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \frac{1}{2} \cdot \sqrt{\frac{1-p}{p}} \cdot \frac{1}{(1-p)^2} \cdot \sqrt{\pi}.$$

$$V_3 = B(p) \cdot p \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right)^2 \cdot \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \frac{1}{2} \cdot \sqrt{\frac{1-p}{p}} \cdot \frac{1}{(1-p)^2} \cdot \sqrt{\pi}.$$

So that $\phi(x)$ is the derivative function of $\Phi(x)$. The condition:

$$\frac{\partial F(p)}{\partial p} = 0$$

is met if $V_1 + V_2 + V_3 = 0$. Since $V_2 > 0$ and $V_3 > 0$ for all p values (according to the assumptions $(1+s) \cdot (1+r)/(1+i) > 1$), and $V_1 < 0$ (because $\partial B(p)/\partial p < 0$), it is theoretically possible that a maximum efficiency value exists. However, the results indicate that V_1 can be a relatively small value compared to V_2 and V_3 if the size of the insurance portfolio is not small, therefore in relatively large insurance portfolios $V_1 + V_2 + V_3 > 0$. If $V_1 + V_2 + V_3 > 0$, then the efficiency measure $F(p)$ increases if p increases, and it can be interpreted so that it is not possible that an insurance portfolio with low-probability high-impact risks belongs to the maximum efficiency (on the contrary, a similar portfolio would belong to a relatively low level of efficiency).

Fig. 1 demonstrates the relationship between the efficiency measure (the ratio between policy-level profitability and insolvency probability) and the insurance event probability for different insurance portfolio sizes ($B(p) = 1 - p^2$, $s = 0.1$, $r = 0.001$, $i = 0$). It is illustrated by Fig. 1 that a maximum efficiency (that belongs to insurance policies with $p < 1$) exists in certain cases only for relatively small insurance portfolios. It is also worth emphasising that the small insurance portfolio that is illustrated by Fig. 1 ($n = 3$ in the example) is associated with a relatively large insolvency probability. This solution for maximum efficiency might therefore be problematic from a practical point of view, when possible solvency requirements are also taken into account.

It is also worth noting that:

$$V_1 + V_2 = \Phi \left(- \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \sqrt{\frac{\pi \cdot p}{1-p}} \right) \cdot \left(\frac{(1+s) \cdot (1+r)}{1+i} - 1 \right) \cdot \left(p \cdot \frac{\partial B(p)}{\partial p} + B(p) \right),$$

and $V_1 + V_2 < 0$ only if:

$$p > - \frac{B(p)}{\frac{\partial B(p)}{\partial p}}.$$

This result indicates that there is also a lower bound for the insurance event occurrence probability so that a maximum efficiency value can exist (so that $p < 1$). It is questionable whether a low-probability high-impact risk corresponds to this lower bound requirement. The exact form of this lower bound of course depends on the definition of the $B(p)$ function. Theoretically, additional assumptions about reinsurance contracts could modify the $B(p)$ function, meaning that the optimal efficiency level could in theory belong to an insurance portfolio with a relatively low probability and high impact (sum insured). However,

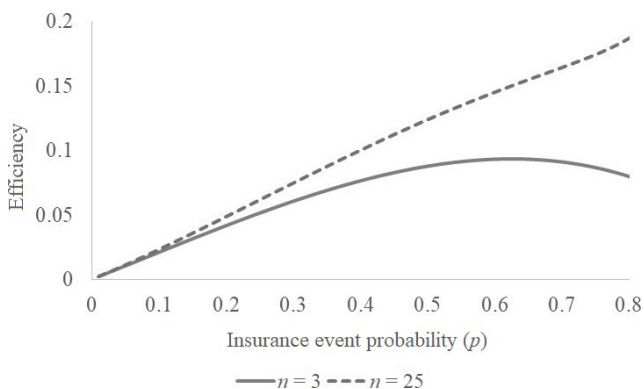


Fig. 1 Efficiency and insurance portfolio size (Source: own calculations)

even in the relatively simple stylised model several factors impede the availability of this result.

On the whole, as a conclusion to the question of whether there exists an efficiency maximising insurance risk level, the answer could be that the conditions for it are quite restrictive. Firstly, the size of the insurance portfolio should be relatively small. Although it may seem easy to assume that a small insurance portfolio is less problematic to build than a larger portfolio, but insurance portfolio size also influences solvency, and a small insurance portfolio may exhibit large insolvency risk. Therefore, it would be problematic to assume that the size of the insurance portfolio is small, and although this paper focuses on another aspect of risk, it would probably not be an acceptable assumption from the point of view of solvency requirements. Secondly, another model result is that there is a theoretical lower bound for the insurance event occurrence probability, and it is questionable whether (apart from the question whether the insurance portfolio size is small enough) this probability belongs to a low-probability high-impact event.

5 Conclusions

Climate change may contribute to an increase in the frequency of low-probability high-impact events. These events can severely influence everyday life and the economy, and if possible, these catastrophes should be avoided. However, it is not always feasible, and then insurance for these low-probability high-impact risks could alleviate at least the financial burden. The lack of catastrophe insurance may have many reasons, and this paper aims to highlight one of these problems: the insurance for low-probability high-impact risk can have inefficient risk and profitability features. This result is based only on a comparison of institution-level risk and profitability, and does not depend on assumptions about the form of utility functions.

This possible problem is examined in a stylised insurance model in which a key element is an elementary feature of low-probability high-impact events: the inverse relationship between the insurance risk (measured by the insurance event probability) and the sum insured (that may indicate the possible impact of the risk). Efficiency is defined as the ratio between profitability and institution-level risk.

Theoretical results suggest that an insurance risk level with maximum efficiency may exist, although only in relatively small insurance portfolios. Since catastrophic events (compared to other types of insurance events) are

relatively rare, the size of an insurance portfolio consisting of similar low-probability high-impact risks could be assumed to be small, but the findings also suggest that there is a lower bound for the insurance risk level that may belong to this optimum. On the whole, it may be concluded that it can be problematic to fulfil the existence conditions for an efficiency optimising insurance risk level, and it is also questionable whether this optimal insurance risk belongs to a low-probability high-impact risk. These findings can add to previous literature by providing a new insight into the reasons for the lack of widespread insurance solutions for catastrophic risks.

References

- Artzner, P., Delbaen, F., Eber, J.-M., Heath, D. (1999) "Coherent Measures of Risk", *Mathematical Finance*, 9(3), pp. 203–228.
<https://doi.org/10.1111/1467-9965.00068>
- Backus, D. K., Gregory, A. W. (1993) "Theoretical Relations Between Risk Premiums and Conditional Variances", *Journal of Business & Economic Statistics*, 11(2), pp. 177–185.
<https://doi.org/10.2307/1391369>
- Bayraktar, E., Milevsky, M. A., Promislow, S. D., Young, V. R. (2009) "Valuation of mortality risk via the instantaneous Sharpe ratio: Applications to life annuities", *Journal of Economic Dynamics and Control*, 33(3), pp. 676–691.
<https://doi.org/10.1016/j.jedc.2008.09.004>
- Bignozzi, V., Macci, C., Petrella, L. (2018) "Large deviations for risk measures in finite mixture models", *Insurance: Mathematics and Economics*, 80, pp. 84–92.
<https://doi.org/10.1016/j.insmatheco.2018.03.005>
- Blanchet, J., Lam, H. (2013) "A heavy traffic approach to modeling large life insurance portfolios", *Insurance: Mathematics and Economics*, 53(1), pp. 237–251.
<https://doi.org/10.1016/j.insmatheco.2013.04.011>
- Bohnert, A., Gatzert, N., Jørgensen, P. L. (2015) "On the management of life insurance company risk by strategic choice of product mix, investment strategy and surplus appropriation schemes", *Insurance: Mathematics and Economics*, 60, pp. 83–97.
<https://doi.org/10.1016/j.insmatheco.2014.11.003>
- Burnecki, K., Giuricich, M. N., Palmowski, Z. (2019) "Valuation of contingent convertible catastrophe bonds – The case for equity conversion", *Insurance: Mathematics and Economics*, 88, pp. 238–254.
<https://doi.org/10.1016/j.insmatheco.2019.07.006>
- Charpentier, A., Le Maux, B. (2014) "Natural catastrophe insurance: How should the government intervene?", *Journal of Public Economics*, 115, pp. 1–17.
<https://doi.org/10.1016/j.jpubeco.2014.03.004>
- Chen, L., Lin, L., Lu, Y., Parker, G. (2017) "Analysis of survivorship life insurance portfolios with stochastic rates or return", *Insurance: Mathematics and Economics*, 75, pp. 16–31.
<https://doi.org/10.1016/j.insmatheco.2017.04.001>
- Cont, R., Deguest, R., Scandolo, G. (2010) "Robustness and sensitivity analysis of risk measurement procedures", *Quantitative Finance*, 10(6), pp. 593–606.
<https://doi.org/10.1080/14697681003685597>
- Cummins, J. D., Doherty, N., Lo, A. (2002) "Can insurers pay for the "big one"? Measuring the capacity of the insurance market to respond to catastrophic losses", *Journal of Banking & Finance*, 26(2–3), pp. 557–583.
[https://doi.org/10.1016/s0378-4266\(01\)00234-5](https://doi.org/10.1016/s0378-4266(01)00234-5)
- Dickson, D. C. M., Hardy, M. R., Waters, H. R. (2011) "Actuarial Mathematics for Life Contingent Risks", Cambridge University Press. ISBN 978-0-521-11825-5
<https://doi.org/10.1017/CBO9780511800146>
- Dietz, S. (2011) "High impact, low probability? An empirical analysis of risk in the economics of climate change", *Climatic Change*, 108(3), pp. 519–541.
<https://doi.org/10.1007/s10584-010-9993-4>
- Duncan, J., Myers, R. J. (2000) "Crop Insurance Under Catastrophic Risk", *American Journal of Agricultural Economics*, 82(4), pp. 842–855.
<https://doi.org/10.1111/0002-9092.00085>
- Embrechts, P., Klüppelberg, C., Mikosch, T. (1997) "Modelling Extremal Events for Insurance and Finance", Springer-Verlag Berlin Heidelberg. ISBN 978-3-540-60931-5
<https://doi.org/10.1007/978-3-642-33483-2>
- Fama, E. F., French, K. R. (2004) "The Capital Asset Pricing Model: Theory and Evidence", *Journal of Economic Perspectives*, 18(3), pp. 25–46.
<https://doi.org/10.1257/0895330042162430>
- Gibson, R., Habib, M. A., Ziegler, A. (2014) "Reinsurance or securitization: The case of natural catastrophe risk", *Journal of Mathematical Economics*, 53, pp. 79–100.
<https://doi.org/10.1016/j.jmateco.2014.05.007>
- Goda, K., Hong, H. P. (2008) "Application of cumulative prospect theory: Implied seismic design preference", *Structural Safety*, 30(6), pp. 506–516.
<https://doi.org/10.1016/j.strusafe.2007.09.007>

The theoretical findings highlight that if only some basic insurance features are modelled, the scarcity of insurance market solutions for catastrophic risks is quite understandable. Obviously, several other considerations could alter the conclusion that insurance for low-probability high-impact risks is not efficient. For instance, reinsurance can theoretically modify the assumed relationship between the event probability and the sum insured, and effects of a heterogeneous insurance product mix could also influence the results. These considerations provide avenues for future research about the optimal role of insurance in the management of catastrophic risks.

- Golubin, A. Y. (2006) "Optimal decision rule in forming an insurance portfolio", *Operations Research Letters*, 34(3), pp. 316–322.
<https://doi.org/10.1016/j.orl.2005.05.011>
- Goodspeed, T. J., Haughwout, A. F. (2012) "On the optimal design of disaster insurance in a federation", *Economics of Governance*, 13(1), pp. 1–27.
<https://doi.org/10.1007/s10101-011-0103-5>
- Grechuk, B., Zabaranin, M. (2014) "Risk averse decision making under catastrophic risk", *European Journal of Operational Research*, 239(1), pp. 166–176.
<https://doi.org/10.1016/j.ejor.2014.04.042>
- Hagendorff, B., Hagendorff, J., Keasey, K. (2015) "The Impact of Mega-Catastrophes on Insurers: An Exposure-Based Analysis of the U.S. Homeowners' Insurance Market", *Risk Analysis*, 35(1), pp. 157–173.
<https://doi.org/10.1111/risa.12252>
- Hofer, L., Zanini, M. A., Gardoni, P. (2020) "Risk-based catastrophe bond design for a spatially distributed portfolio", *Structural Safety*, 83, 101908.
<https://doi.org/10.1016/j.strusafe.2019.101908>
- Ibragimov, R., Jaffee, D., Walden, J. (2009) "Nondiversification Traps in Catastrophe Insurance Markets", *The Review of Financial Studies*, 22(3), pp. 959–993.
<https://doi.org/10.1093/rfs/hhn021>
- Ibragimov, R., Walden, J. (2007) "The limits of diversification when losses may be large", *Journal of Banking & Finance*, 31(8), pp. 2551–2569.
<https://doi.org/10.1016/j.jbankfin.2006.11.014>
- Insurance Europe (2014) "Why insurers differ from banks", [pdf] Insurance Europe, Brussels, Belgium. Available at: <file:///C:/Users/user/Downloads/Why%20insurers%20differ%20from%20banks.pdf> [Accessed: 16 April 2021]
- Khan, M. A., Sun, Y. (2003) "Exact arbitrage, well-diversified portfolios and asset pricing in large markets", *Journal of Economic Theory*, 110(2), pp. 337–373.
[https://doi.org/10.1016/S0022-0531\(03\)00038-3](https://doi.org/10.1016/S0022-0531(03)00038-3)
- Kousky, C., Cooke, R. (2012) "Explaining the Failure to Insure Catastrophic Risks", *The Geneva Papers on Risk and Insurance - Issues and Practice*, 37(2), pp. 206–227.
<https://doi.org/10.1057/gpp.2012.14>
- Kunreuther, H. (1996) "Mitigating disaster losses through insurance", *Journal of Risk and Uncertainty*, 12(2), pp. 171–187.
<https://doi.org/10.1007/BF00055792>
- Kunreuther, H., Pauly, M. (2004) "Neglecting Disaster: Why Don't People Insure Against Large Losses?", *Journal of Risk and Uncertainty*, 28(1), pp. 5–21.
<https://doi.org/10.1023/B:RISK.0000009433.25126.87>
- Lamond, J., Penning-Rowsell, E. (2014) "The robustness of flood insurance regimes given changing risk resulting from climate change", *Climate Risk Management*, 2, pp. 1–10.
<https://doi.org/10.1016/j.crm.2014.03.001>
- Lintner, J. (1965) "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", *The Review of Economics and Statistics*, 47(1), pp. 13–37.
<https://doi.org/10.2307/1924119>
- Louaas, A., Picard, P. (2018) "Optimal insurance coverage of low probability-high severity risks", [preprint] HAL, hal-01924408, 15 November. [online] Available at: <https://hal-polytechnique.archives-ouvertes.fr/hal-01924408> [Accessed: 16 April 2021]
- Lundblad, C. (2007) "The risk return tradeoff in the long run: 1836–2003", *Journal of Financial Economics*, 85(1), pp. 123–150.
<https://doi.org/10.1016/j.jfineco.2006.06.003>
- Luo, S., Taksar, M., Tsoi, A. (2008) "On reinsurance and investment for large insurance portfolios", *Insurance: Mathematics and Economics*, 42(1), pp. 434–444.
<https://doi.org/10.1016/j.insmatheco.2007.04.002>
- Markowitz, H. (1952) "Portfolio selection", *The Journal of Finance*, 7(1), pp. 77–91.
<https://doi.org/10.1111/j.1540-6261.1952.tb01525.x>
- Marks, J. M., Nam, K. (2018) "Intertemporal risk-return tradeoff in the short-run", *Economics Letters*, 172, pp. 81–84.
<https://doi.org/10.1016/j.econlet.2018.08.031>
- Merton, R. C. (1973) "An Intertemporal Capital Asset Pricing Model", *Econometrica*, 41(5), pp. 867–887.
<https://doi.org/10.2307/1913811>
- Münchener Rückversicherungs-Gesellschaft, NatCatSERVICE (2022) "Munich Re NatCatSERVICE: Natural catastrophes in 2021", [pdf] Münchener Rückversicherungs-Gesellschaft, NatCatSERVICE. [online] Available at: https://www.munichre.com/content/dam/munichre/mrwebsiteslaunche/natcat-2022/2021_Figures-of-the-year.pdf/_jcr_content/renditions/original/2021_Figures-of-the-year.pdf [Accessed: 19 April 2022]
- Robinson, P. J., Botzen, W. J. W. (2019) "Economic experiments, hypothetical surveys and market data studies of insurance demand against low-probability/high-impact risks: a systematic review of designs, theoretical insights and determinants of demand", *Journal of Economic Surveys*, 33(5), pp. 1493–1530.
<https://doi.org/10.1111/joes.12332>
- Salvador, E., Floros, C., Arago, V. (2014) "Re-examining the risk–return relationship in Europe: Linear or non-linear trade-off?", *Journal of Empirical Finance*, 28, pp. 60–77.
<https://doi.org/10.1016/j.jempfin.2014.05.004>
- Schram, A., Sonnemans, J. (2011) "How individuals choose health insurance: An experimental analysis", *European Economic Review*, 55(6), pp. 799–819.
<https://doi.org/10.1016/j.euroecorev.2011.01.001>
- Sharpe, W. F. (1964) "Capital asset prices: a theory of market equilibrium under conditions of risk", *The Journal of Finance*, 19(3), pp. 425–442.
<https://doi.org/10.1111/j.1540-6261.1964.tb02865.x>
- Subramanian, A., Wang, J. (2018) "Reinsurance versus securitization of catastrophe risk", *Insurance: Mathematics and Economics*, 82, pp. 55–72.
<https://doi.org/10.1016/j.insmatheco.2018.06.006>
- Tsai, J. T., Wang, J. L., Tzeng, L. Y. (2010) "On the optimal product mix in life insurance companies using conditional value at risk", *Insurance: Mathematics and Economics*, 46(1), pp. 235–241.
<https://doi.org/10.1016/j.insmatheco.2009.10.006>

- Vendrame, V., Tucker, J., Guermat, C. (2016) "Some extensions of the CAPM for individual assets", *International Review of Financial Analysis*, 44, pp. 78–85.
<https://doi.org/10.1016/j.irfa.2016.01.010>
- Wang, J. L., Huang, H. C., Yang, S. S., Tsai, J. T. (2010) "An Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies: The Immunization Theory Approach", *The Journal of Risk and Insurance*, 77(2), pp. 473–497.
<https://doi.org/10.1111/j.1539-6975.2009.01325.x>
- Wang, J., Yang, M. (2013) "On the risk return relationship", *Journal of Empirical Finance*, 21, pp. 132–141.
<https://doi.org/10.1016/j.jempfin.2013.01.001>
- Wirtz, A., Kron, W., Löw, P., Steuer, M. (2014) "The need for data: natural disasters and the challenges of database management", *Natural Hazards*, 70(1), pp. 135–157.
<https://doi.org/10.1007/s11069-012-0312-4>
- Wu, Y.-C. (2015) "Reexamining the feasibility of diversification and transfer instruments on smoothing catastrophe risk", *Insurance: Mathematics and Economics*, 64, pp. 54–66.
<https://doi.org/10.1016/j.insmatheco.2015.04.007>
- Wu, Y.-C. (2020) "Equilibrium in natural catastrophe insurance market under disaster-resistant technologies, financial innovations and government interventions", *Insurance: Mathematics and Economics*, 95, pp. 116–128.
<https://doi.org/10.1016/j.insmatheco.2020.08.006>
- Yang, M. (2019) "The risk return relationship: Evidence from index returns and realised variances", *Journal of Economic Dynamics and Control*, 107, 103732.
<https://doi.org/10.1016/j.jedc.2019.103732>
- Zhao, J., Lee, J. Y., Li, Y., Yin, Y.-J. (2020) "Effect of catastrophe insurance on disaster-impacted community: Quantitative framework and case studies", *International Journal of Disaster Risk Reduction*, 43, 101387.
<https://doi.org/10.1016/j.ijdrr.2019.101387>