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STRUCTURAL THERMO-MECHANICS

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Abstract

A method has been elaborated for the dimensioning and design of heat isulated vessels, through which the mechanical element can be given thermo-dynamical and the thermo-technical element can be given strength function. In the case of plane-walled vessels we achieved a rather favourable design. Vessels dimensioned in this way contain considerably less steel and have a more advantageous heat isulation.

A further result is the generalized method elaborated for the optimal design of multi-element, multi-function structures.

Keywords: thermoelasticity, heat transfer.

Introduction

To find the optimal structure is a long-standing and basic ambition. This is even more important if the structure serves various purposes, e. g. strength and thermo-technical ones simultaneously, and it is particularly true if further functions are also involved.

The idea that the isulation should perform static functions, and the mechanical structure thermo-technical ones as well, occurred first with regard to isulated, cylindrical vessels. This called for metal-isulation-metal design. As is shown by the calculation to be presented, this structure did not meet the expectations due to the stress state and the considerable variance between the material characteristics of the metal and the isulation.

This idea, however, proved to be right in the case of plane-walled, heat isulated vessels. Owing to the metal-isulation-metal design here realized, and to the stress state corresponding to the bending, the structure thus resulting from the same amount of material has significantly more favourable static features and is somewhat more advantageous in terms of thermo-technical qualities.

Encouraged by such results, we attempted to generalize the experience related to multi-purpose structures.

Mechanical Dimensioning of Isulated Vessels

With the traditional isulation method — involving metal-isulation arrangement — the isulation has no mechanical role to play. In such cases the dimensioning is also done in the traditional way. With the appropriate design, the metal-isulation-metal arrangement seems to be advantageous, as in this way the isulation participates in carrying the mechanical load.

In the following sections this latter solution will be examined first in the case of cylindrical, then, in the case of plane-walled vessels.

Cylindrical Vessel

Let us examine the conditions with the arrangement and indications corresponding to *Fig. 1*. The problem and solution of multi-layer vessels joint with overlaps is widely known and dealt with in the technical literature, e.g. [1], a brief summary is provided here only to draw the necessary conclusions.

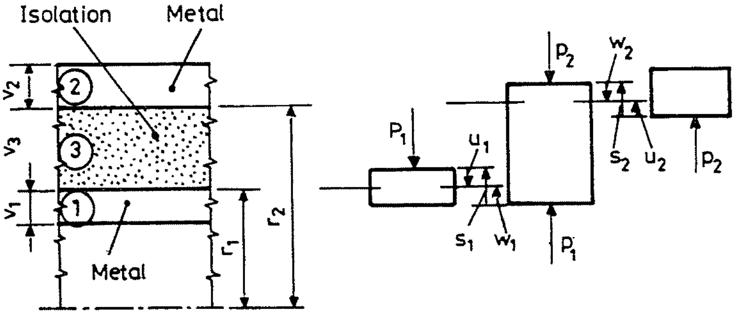

Fig. 1. Overlaps in cylindrical vessel

Introducing the matrices $\mathbf{s}^* = [s_1, s_2]$ which summarizes the overlaps at two joining points, and $\mathbf{u}^* = [u_1, u_2]$ and $\mathbf{w}^* = [w_1, w_2]$ summarizing the displacements at the joining points, the equation

$$\mathbf{s} = \mathbf{u} + \mathbf{w} \tag{1}$$

can be written.

Introducing, furthermore, the matrix $\mathbf{p}^* = [p_1, p_2]$ summarizing the pressures at the joining points and matrices

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{\beta} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta 22 \end{bmatrix}$$

summarizing the spring constant in accordance with the interpretation that

$$\mathbf{u} = \gamma \mathbf{p}\,, \qquad \mathbf{w} = \boldsymbol{\beta} \mathbf{p} \tag{2}$$

based on (1) and (2)

$$\mathbf{s} = (\boldsymbol{\gamma} + \boldsymbol{\beta}) \cdot \mathbf{p} \tag{3/I}$$

and

$$\mathbf{p} = (\boldsymbol{\gamma} + \boldsymbol{\beta})^{-1} \cdot s \tag{3/II}$$

can be written where γ_i and β_{ij} are the displacement that belongs to a unit of pressure in thin- or thick-walled tube.

Thus, knowing the overlaps (s), the stresses (p) can be calculated in accordance with (3/II), and on the basis of this the revision as well as the dimensioning of the individual layers can be performed, and only the following considerations need to be taken into account as a supplement:

It cannot be determined in advance which operating state is the least advantageous one — whether it is when the structure is in its initial assembled state without any load, or it is the actual operating state (v) when there is internal pressure (p) and heating (m) as well, or maybe a subcase of this latter when one of the two is zero. Accordingly, instead of (3/II)

$$\mathbf{p}_v = (\boldsymbol{\gamma} + \boldsymbol{\beta})^{-1} \cdot s_v \tag{3}$$

holds true where

$$\mathbf{s}_v = \mathbf{s} + \mathbf{s}^p + \mathbf{s}^m\,. \tag{4}$$

Here

$$\mathbf{s}^p = \boldsymbol{\gamma} \mathbf{p}_{\ddot{u}}\,, \qquad \mathbf{p}^*_{\ddot{u}} = [p_{\ddot{u}}, 0]\,, \tag{5}$$

where $p_{\ddot{u}}$ is the working pressure;

$$\mathbf{s}^m = \begin{bmatrix} r_1 \alpha \Delta t_1 - \Delta u_3^b \\ -r_2 \alpha \Delta t_2 + \Delta u_3^k \end{bmatrix} \tag{6}$$

is the component taking into account the warming during operation. α here is the coefficient of thermal expansion, Δt_i is the temperature change of the individual materials, and Δu_3^b and Δu_3^k are the heat expansion of the isulation outside and inside.

The calculations thus carried out show that due to the significant difference in the Young modulus of the steel and the isulation, the structure behaves in an unexpected way, in the case of the cylindrical vessels the metal-isulation-metal arrangement does not have the desired advantages.

Plane-Walled Vessel

Let us assume that the only kind of load in the vessel wall is bending and let us disregard shearing. Using the indications of *Fig. 2,* let us compare the maximum stress generated in the metal part of the metal-isulation and of the metal-isulation-metal arrangement.

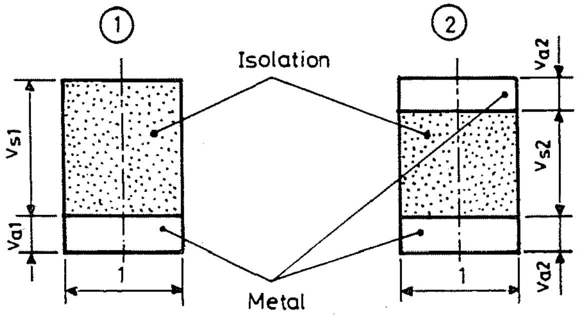

Fig. 2. The metal-isulation and metal-isulation-metal arrangement

We can write that in case ①

$$\sigma^{①} = \frac{M}{I^{①}} \cdot e^{①}, \qquad I^{①} = \frac{1}{12} \cdot v_{a1}^{3}, \qquad e^{①} = \frac{v_{a1}}{2},$$

that is

$$\sigma^{①} = 6 \cdot \frac{M}{v_{a1}^{2}}. \tag{7}$$

In case ②, however,

$$\sigma^{②} = \frac{M}{I^{②}} \cdot e^{②}, \qquad I^{②} = v_{a2}^{3} \left(\frac{4}{6} + \frac{v_{s2}}{v_{a2}} + \frac{1}{2} \cdot \frac{v_{s2}^{2}}{v_{a2}^{2}} \right),$$

$$e^{②} = v_{a2} \left(\frac{v_{s2}}{2v_{a2}} + 1 \right),$$

that is

$$\sigma^{②} = \frac{M}{v_{a2}^{3} \left(\frac{4}{6} + \frac{v_{s2}}{v_{a2}} + \frac{1}{2} \cdot \frac{v_{s2}^{2}}{v_{a2}^{2}} \right)} \cdot v_{a2} \left(\frac{v_{s2}}{2v_{a2}} + 1 \right). \tag{8}$$

(7) and (8) were based on the assumption that in case ② the metal and the isulation work together whereas in case ① they do not.

Let us equalize $\sigma^{①}$ and σ^2 and let us then reduce the relation received! In the end we receive that

$$\frac{2v_{a2}}{v_{a1}} = \sqrt{\frac{1+\frac{1}{2}\cdot\frac{v_{s2}}{v_{a2}}}{1+\frac{3}{2}\cdot\frac{v_{s2}}{v_{a2}}+\frac{3}{4}\cdot\frac{v_{s2}^2}{v_{a2}^2}}}\,. \tag{9}$$

This relation shows how the quantity of metal to be used decreases as a function of the thickness of the isulation (See *Fig. 3*).

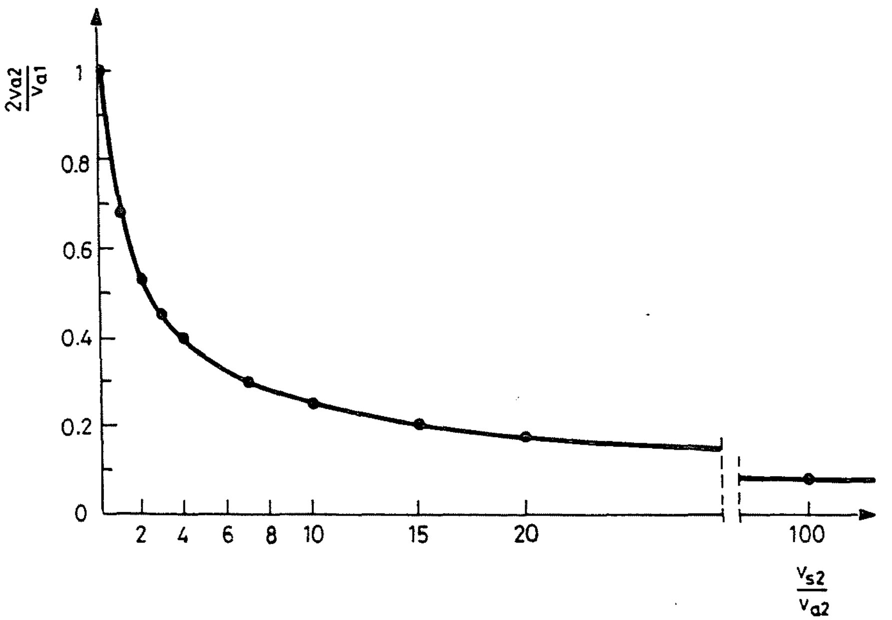

Fig. 3. Quantity of metal as a function of the tickness

Mechanical and Thermodynamical Dimensioning of Isulated Vessel

So far isulation has only been taken into consideration in the strength dimensioning, thermo-technical relations were not taken into account. Let us now turn our attention to this aspect.

In the case of a layered wall, heat isulation is characterized by the coefficient of heat transmission:

$$k = \frac{1}{\frac{1}{\alpha_1} + \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_2}} \tag{10/I}$$

where α_i is the coefficient of heat transfer on the outside surfaces, λ_i and δ_i are the coefficient of heat conduction and thickness of the individual layers, respectively, and n is the number of layers. The expression of k needs completion, as due to the metal-isulation-metal structure, some fictive coefficient of heat transfer must appear between the individual layers, since heat between the layers is not propagated by way of heat conduction purely. Accordingly

$$k = \frac{1}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} + \sum_{j=1}^{n-1} \frac{1}{\alpha_j^*}} \tag{10}$$

where α_j^* is the fictive coefficient of heat transfer between the layers. (10) clearly shows the thermo-dynamical advantages of the multi-layer structure.

Consequently, in the case of the two different structures shown in *Fig. 2*, the method of dimensioning can be as follows. In case ①, the mechanical and thermodynamical elements work separately and perform the two different functions independently of each other. In case ②, however, they work and perform the two tasks together. The strength criterion is that the maximum stress generated in the two cases should be equal while the thermodynamical condition is the equality of the coefficients of heat transmission in the two cases.

Obviously, these criteria cannot be satisfied simultaneously, however, their fulfilment is not essential. In case ①, thus, based on $\sigma^{①}$ and $k^{①}$, v_{a1} and v_{s1} result. In case ②, v_{a2} results through relation (9) or through the diagram in *Fig. 3* if we assume that $v_{s2} \approx v_{s1}$. In this, case $\sigma^{②} = \sigma^{①}$ and $k^{②} = \mu k^{①}$ where $\mu = 0.9 \div 1.0$, i. e. the heat isulation is more favourable to some extent as well.

Some Aspects of the Design of Multi-Element, Multi-Function Structures

Let us attempt to generalize the thought outlined above. Our starting point should be the table in *Fig. 4.* A structure with various elements for its various functions is given. Let us characterize the extent to which the elements participate in the various functions by numbers ranging from 0 to 1. In our specific case, for example, if the mechanical element has only functions strength and the thermotechnical element has only thermodynamical ones, the main diagonal of the table contains only 1s and zero appears in all other places. This is obviously, the least advantageous case.

Element \ Function	Strength	Thermodynamical
Mechanical		
Thermotechnical		

Fig. 4. Elements and functions in a structure

Let us now, for the sake of further examinations, introduce the F $n\times1$, so-called function matrix, which shows what kind of tasks the structure has, the E $m\times1$, the so-called element matrix, which shows the elements of the structure and the $\mathbf{P}$ $n\times m$, the so-called measure matrix, which shows which element to what extent performs the appropriate function. The elements of F and E are unit and the total of row vectors of $\mathbf{P}$ is 1. Thus, the following is true that

$$F = \mathbf{P}E. \tag{11}$$

The value, that is the rentability of the structure is characterized by $\mathbf{P}$. Let us introduce the $\mathbf{P}..\mathbf{P}$ double scalar product as measurement. Min($\mathbf{P}..\mathbf{P}$) gives the ideal solution, a good approximation of which should be aimed at. It is easy to understand, that in case 2×2 of the table in *Fig. 4*, the ideal solution is, if all elements of $\mathbf{P}$ are 0.5. In this case $\mathbf{P}..\mathbf{P} = 1$ and in the least advantageous case $\mathbf{P}..\mathbf{P} = 2$. Thus, the ideal case is if every element serves each function equally, the least favourable case is if each element serves only its own function. The same can be understood in the case of 3×3 $\mathbf{P}$. In this case $\min(\mathbf{P}..\mathbf{P}) = 1$ and the least favourable $\mathbf{P}..\mathbf{P} = 3$.

Summary

Based on the investigations outlined, a method has been elaborated for the design of plane-walled, heat isulated warm water vessels by which the use of considerably less steel became possible and heat isulation also proved to be more advantageous. On the other hand, however, this involves a costlier production technology.

A further result is the generalization of the process, a method by which multi-purpose, multi-element structures can be examined.

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