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THE STARTING SECTION OF PNEUMATIC CONVEYING

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Abstract

In the starting section of a pneumatic conveyor the material accelerates from the zero starting velocity to the velocity of the uniform conveyance section. In the case of thin flow conveying the pressure drop needed for material acceleration is of a considerable value compared with other pressure drops. In the case of short (15 to 25 m) conveying it forms the greatest part of additional pressure drops. Furthermore, in a vertical starting section the pressure drop needed to lift the weight of the material is higher than in the uniform velocity section.

Keywords: pneumatic conveying.

1. Introduction

The pressure drop arising during conveying is an operating parameter of pneumatic conveyors. Calculation methods for determining the pressure drop relying on measurements done on trial and industrial equipment and supported by theoretical considerations have been developed. In most cases these methods only deal with the pressures of the so-called steady-state section disregarding pressure drop in the so-called starting section following the inlet [14].

Figs. 1 and *2* show pressures p along the length ℓ of horizontal and vertical, thin flow pneumatic conveying pipes. It is also seen from these measurement data that a considerable pressure drop results from the acceleration of the solid phase in the so-called starting section ℓ_{inst} following the inlet. After the starting section, in a straight conveying pipe pressure drop is proportional to the pipe length. This is only true for a negligible gas expansion; hence in thin flow conveying.

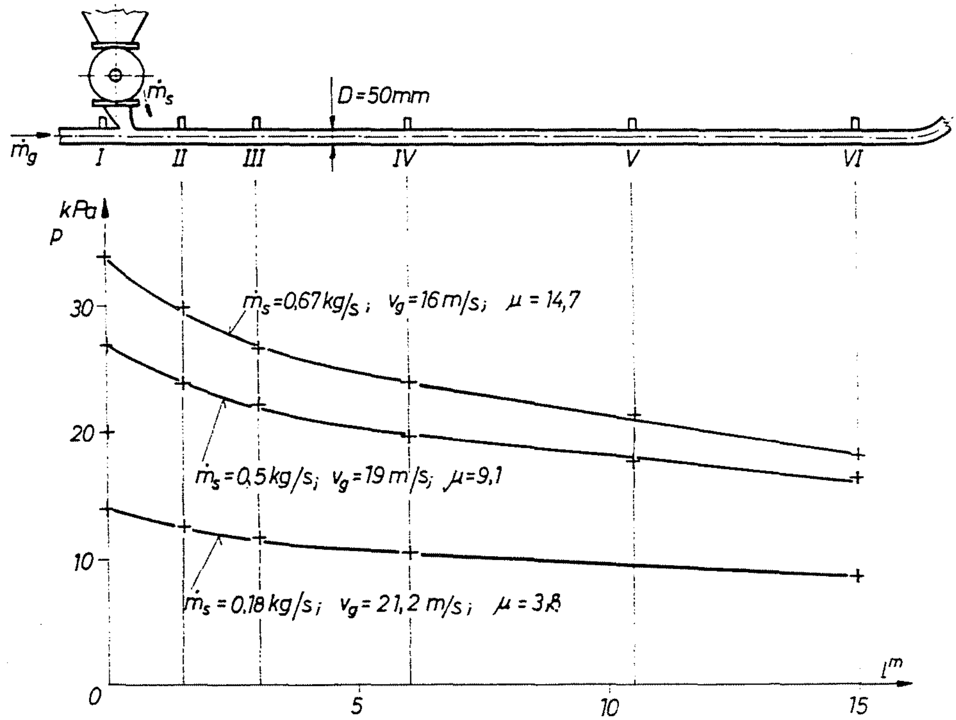

Fig. 1. Pressure p at horizontal pressure-mode conveying of granulated sugar as a function of pipe length ℓ. Diameter of conveying pipe: $D = 50$ mm, I-VI: locations of pressure measurement

2. Interpretation of the Measurement Results

The starting section will be examined separately for thin flow conveying in horizontal and in vertical pipes, based on measurement data.

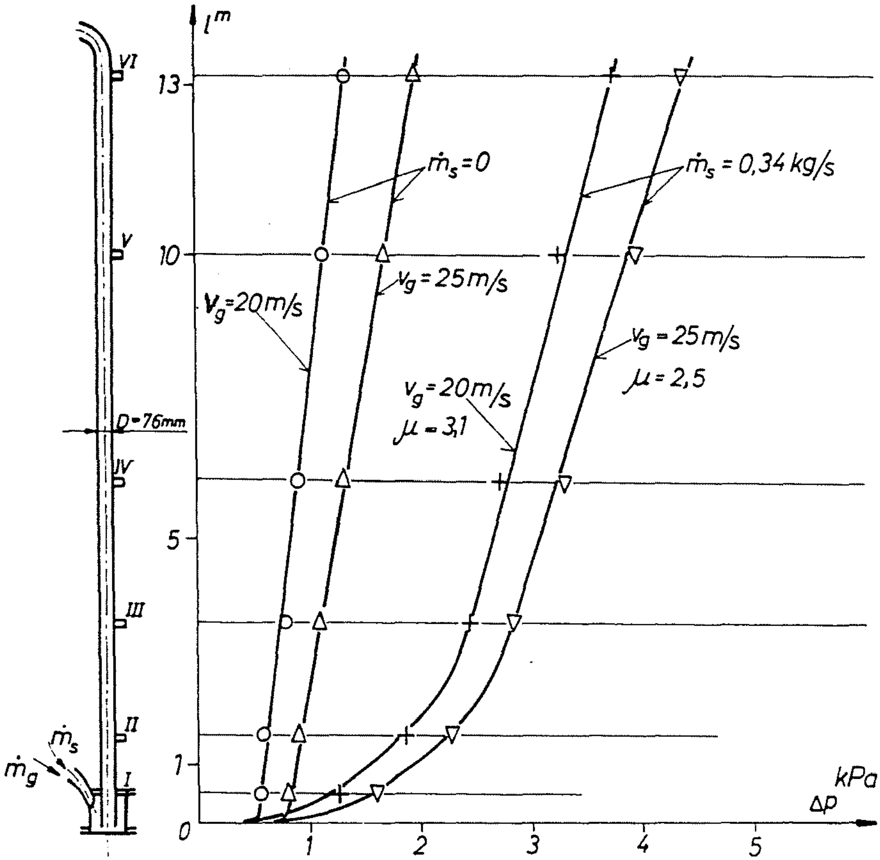

Fig. 2. Pressure drop Δp at vertical suction mode conveying of mill semi-finished product as a function of pipe length $D = 76$ mm

2.1 Horizontal Starting Section

Pressures and velocities of horizontal, suction-mode thin flow conveying as a function of pipe length are seen in *Fig. 3*. Two pressure characteristics have been plotted for a constant amount of gas $\dot{m}_g$.

Δp_0 is the no-load ($\dot{m}_s = 0$) curve for an $\dot{m}_g$ amount of flowing gas. It starts with the inlet pressure drop Δp_b to be followed by the frictional pressure drop of the straight pipe ($\Delta p_0 = \Delta p_b + \Delta p_{p0}$).

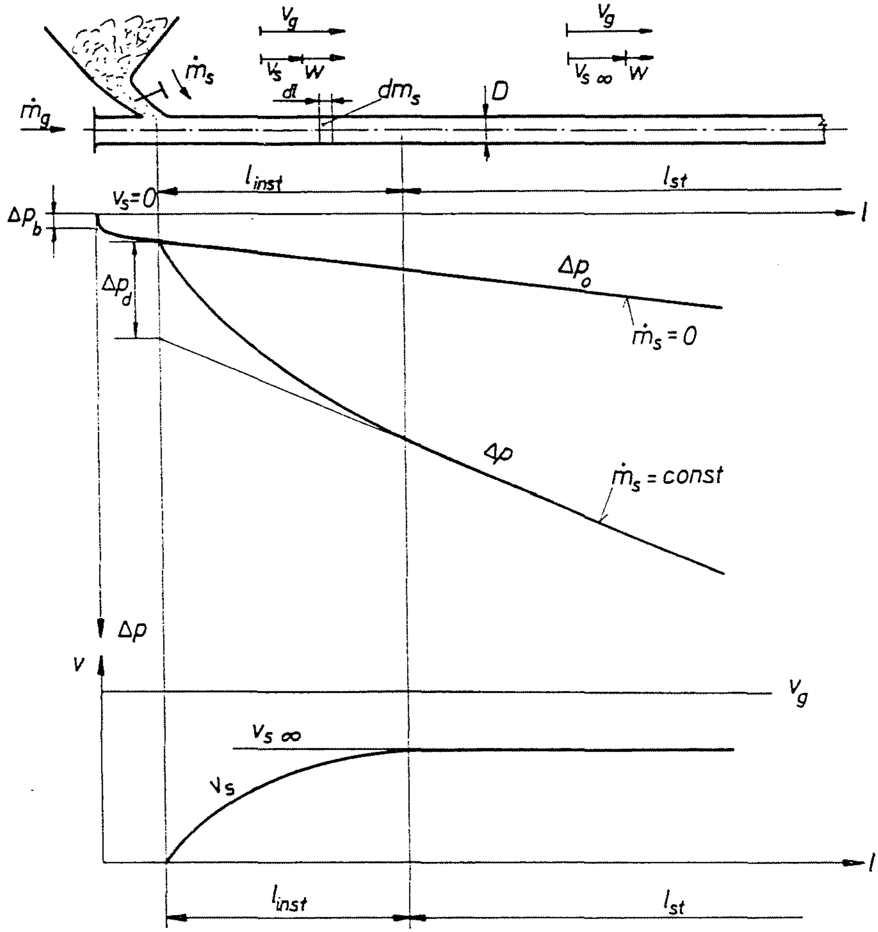

Fig. 3. Pressures p and velocities v in horizontal pipe as a function of pipe length ℓ

The inlet pressure drop is:

$$\Delta p_b = \varrho_g (1 + \zeta_b) v_g^2 / 2, \tag{1}$$

where ζ_b is the inlet loss factor. Its value is in the case of a well rounded pipe: $\zeta_b = 0.05 - 0.3$.

The frictional pressure drop in a straight pipe:

$$\Delta p_{p0} = \lambda \frac{\ell}{D} \frac{v_g^2}{2} \varrho_g, \tag{2}$$

where λ is the pipe friction factor of gas flow. Its value depends on the Reynolds' number and the roughness of the pipe wall. For pneumatic conveyor pipe $\lambda = 0.015 - 0.025$.

The curve marked Δp shows the change of pressures forming during material conveying $\dot{m}_s$ for a gas quantity $\dot{m}_g$ equal to the no-load state. The pressure drop Δp in material conveying may be taken as sum of the so-called additional pressure drop Δp_j and the no-load pressure drop Δp_0:

$$\Delta p = \Delta p_0 + \Delta p_j.$$

In the so-called starting section ℓ_{inst} from the inlet the solid phase accelerates from velocity $v_s = 0$ to the velocity of the steady section $v_{s\infty}$, so the pressure gradient (pressure change per unit length, $\mathrm{d}p/\mathrm{d}l$) exceeds that in the steady-state conveying section ℓ_{st} after the starting section, to be considered as steady-state flow.

There is a constant pressure gradient in the steady-state conveying section ($\mathrm{d}p/\mathrm{d}l$ = const.), which means that pressure varies linearly with the pipe length. The value of material velocity is constant (see *Fig. 3*).

As it will be seen from the detailed analysis, material velocity is asymptotically approximating the value of the steady section $v_{s\infty}$ and reaches it in fact in infinite time, hence, along an infinite path. The material acceleration is considered as finished when it has approximated the final value 5%. The length of the starting section ℓ_{inst} is the path length along the one the steady-state velocity is approximated to 5%.

Detailed measurement and theoretical analysis of the horizontal starting section show that in the starting section the additional pressure drop of material conveyance exceeds the additional pressure drop of the steady section by the pressure drop necessary to accelerate the solid phase Δp_d. *Fig. 3* shows an accelerating pressure drop Δp_d curve plotted in this sense. This is verified by Siegel's measurements [5] seen in *Fig. 4*. Also computed Δp_d have been plotted here, calculated with material velocities involving data $w_0 = 8.4$ m/s (Siegel's data), $k_e = 0.3$ and $k_u = 0.001$ from (21).

The hypothesis above that in the starting section the components beyond the ones accelerating additional pressure drop (that is, collision pressure drop Δp_u and lifting pressure drop Δp_e) invariably follow steady-section values is only an approximation.

This approximation is admissible in a horizontal starting section but it is not for vertical conveying. Namely in horizontal uniform conveying

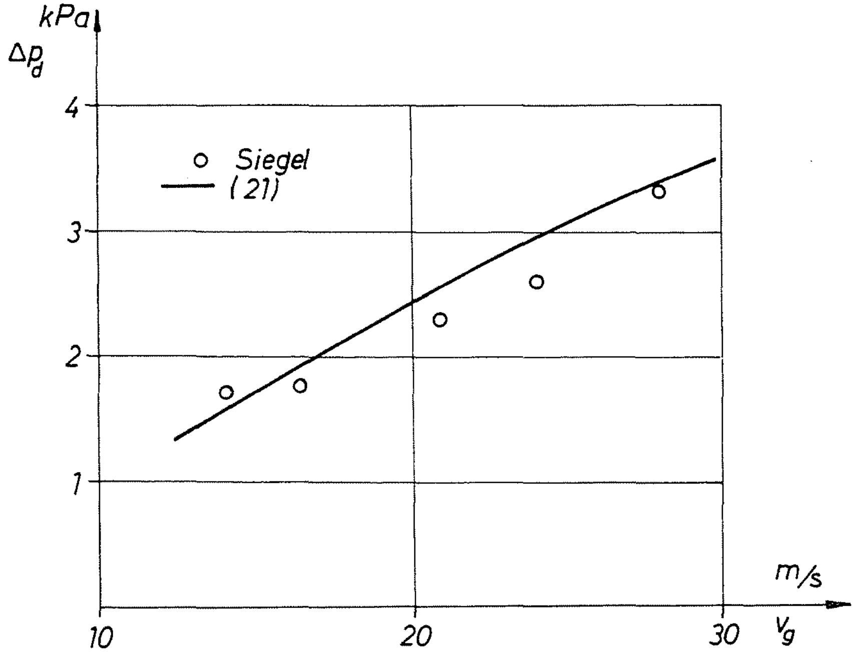

Fig. 4. The values of the accelerating pressure drop measured by SIEGEL [5] and those calculated with *Eq.* (21). Horizontal conveying of wheat, D = 50 mm, $\dot{m}_s$=0.37 kg/s

the two components (Δp_u and Δp_e) are nearly equal, while in the starting section Δp_e increases by about as much as Δp_u decreases. In the case of vertical uniform conveying the lifting pressure drop (Δp_e) considerably exceeds the collision pressure drop. In the starting section, this considerably increased value must not be neglected.

2.2 Vertical Starting Section

Fig. 5 shows pressures and velocities developing in the vertical starting section.

Δp_0 is the curve of the no-load pressure drop, composed — similarly to horizontal conveying — of the Δp_b inlet pressure drop and Δp_{p0} friction pressure drop of the straight pipe.

Δp is the material conveyance pressure drop (plotted for mass flow of gas $\dot{m}_g$ equal to the no-load state) exceeding now also the no-load pressure drop by the so-called additional pressure drop. Thus, additional pressure drop is the pressure between curves Δp and Δp_0. Far from the inlet, in the

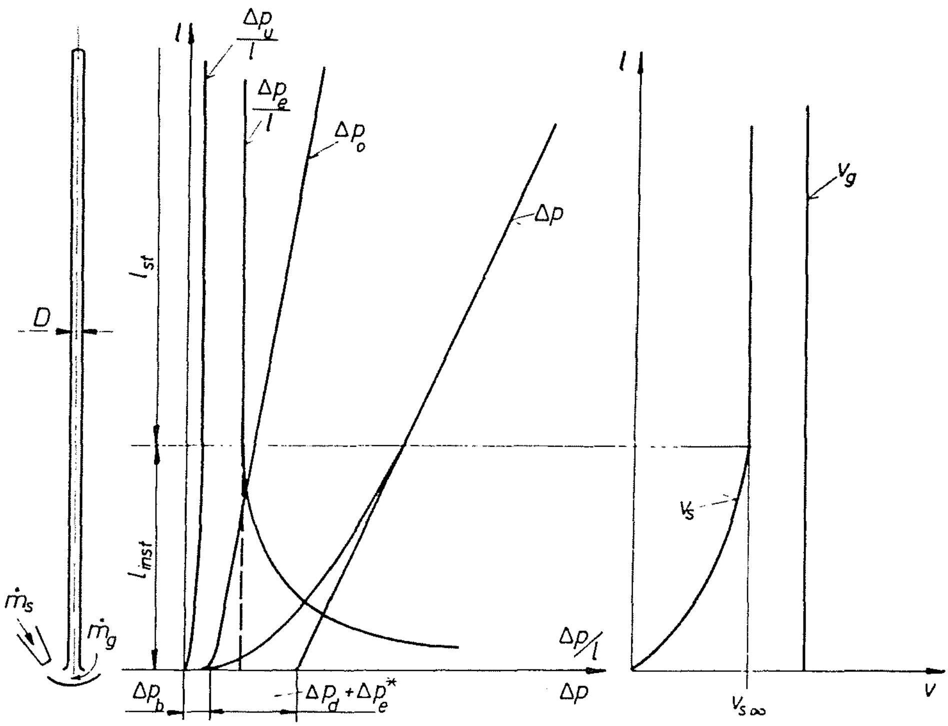

Fig. 5. Pressures and velocities in a vertical pipe

steady section ℓ_{st}, where the material transporting velocity v_s may be taken as constant, the additional pressure drop varies proportionally to the pipe length ($\mathrm{d}p/\mathrm{d}l = \text{const}$). Though, in the starting section ℓ_{inst} the additional pressure drop exceeds that in the steady section by the accelerating pressure drop Δp_d necessary to accelerate the solid phase from velocity $v_s = 0$ to the velocity of the uniform velocity operation $v_{s\infty}$ and by the additional pressure drop Δp_c^* needed to lift the weight of the solid phase.

The $\Delta p_d + \Delta p_c^*$ value has been constructed in *Fig. 5*. The increase of the lifting pressure value Δp_c^* in the starting section can be explained by the following: As the material velocity is smaller in the starting section ℓ_{inst} than in the steady section ($v_s < v_{s\infty}$), there is more material here at the same time than in the steady section of the same length. *Fig. 5* also shows values of lifting pressure drop per unit length $\Delta p_c/l$. In conformity with Item 3, this is inversely proportional to material velocity v_s, thus, its value is greater in the starting section than in the steady section. At the same time, the change of collision pressure drop is negligible per unit length $\Delta p_u/l$ — which is considerably lower than the lifting component already in the steady section.

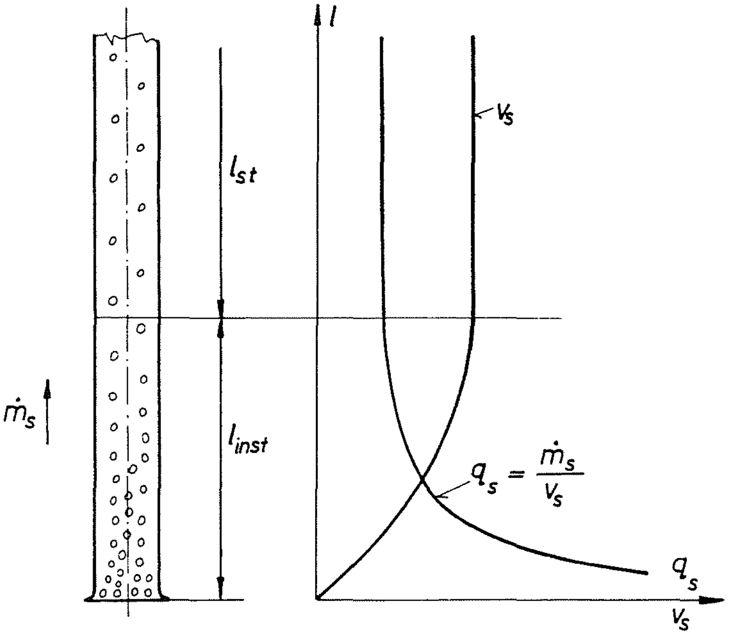

Fig. 6. There is more material in the starting section ℓ_{inst} than in the steady pipe section of the same length ℓ_{st} at the same moment

Fig. 6 shows the consequence of the motion at variable speed of the granular matter in the vertical starting section. There is more material in the low velocity section at the same moment. (There is a greater particle density). The mass of material per one meter q_s is inversely proportional to the matter velocity:

$$q_s = \dot{m}_s/v_s. \tag{3}$$

The mass of the material staying in the starting section in a given moment (and its weight to be balanced by the lifting pressure) exceeds that in a steady operation pipe section of the same length.

In the vertical starting section, the increased pressure drop Δp_i consists of two parts: the pressure drop of acceleration Δp_d and the lifting pressure drop for the additional weight Δp_e^*:

$$\Delta p_i = \Delta p_d + \Delta p_e^*. \tag{4}$$

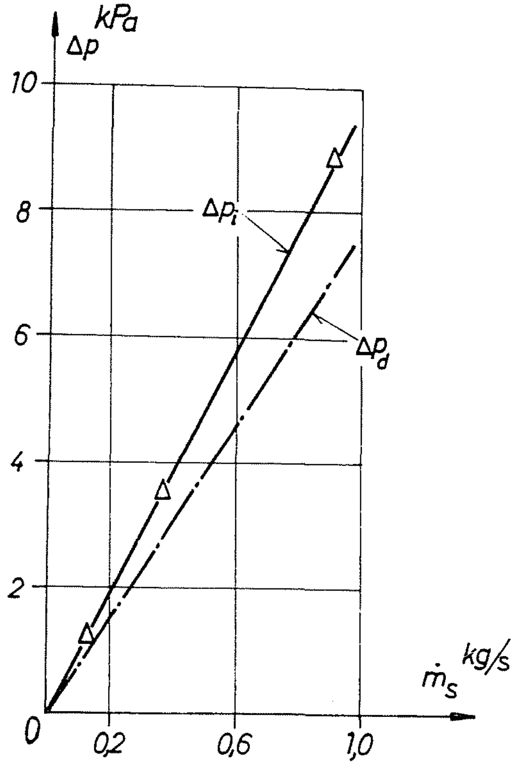

Fig. 7. Additional pressure Δp_i values measured by FLATOW [6] and calculated values of the accelerating pressure drop Δp_d, in a vertical starting section. The difference of the two pressures is the pressure necessary for lifting the excess material of the starting section ($\Delta p_e^* = \Delta p_i - \Delta p_d$). Wheat, $D = 50$ mm, $v_g = 25$ m/s, $v_s = 15.5$ m/s (calculated from (20))

This is also verified by the measurement series of FLATOW [6] (see *Fig. 7*). The values of additional pressures Δp_i forming compared to the stationary section and measured in the vertical starting section are seen in *Fig. 7*. Although Flatow considered the values of Δp_i to be identical to the values of the accelerating pressure drop Δp_d, it can be demonstrated that values of Δp_i are greater than the accelerating pressure drop because of the additional lifting pressure (Δp_d is drawn with dash-and-dot line in the figure).

3. Examination of the Uniform Conveying and the Starting Section Based on the Forces Acting on the Particles

3.1 The Formation of the Additional Pressure Drop

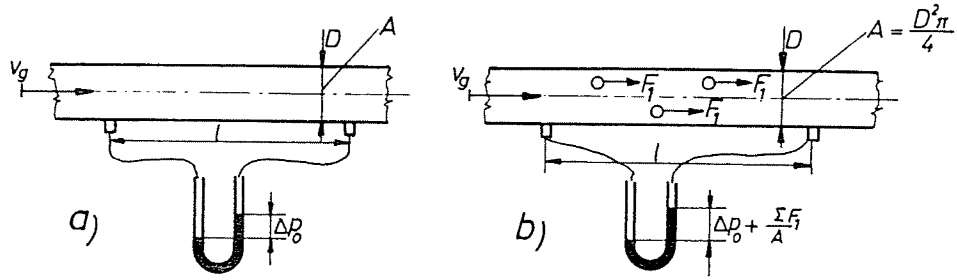

Fig. 8. Formation of the additional pressure drop

The so-called pipe friction pressure drop arising when a homogeneous medium (e.g. gas) is flowing in a straight pipe of D diameter (this is called no-load pressure drop in pneumatic conveying) can be calculated from (2), and measured according to *Fig. 8/a.* If there are bodies in the pipe much smaller than its diameter irrespective of their staying or moving with velocity v_s, a force will act on them due to the flowing gas (aerodynamical force), and, thus, the no-load pressure drop will change. If $v_s < v_g$ and force F_1 is acting on the individual particles, now on the base of superposition, pressure drop in the pipe of A cross-section (*Fig. 8/b*) is:

$$\Delta p = \Delta p_0 + \frac{\Sigma F_1}{A}. \tag{5}$$

In the case of pneumatic conveying the pressure increase is called additional pressure drop Δp_j:

$$\Delta p_j = \frac{\Sigma F_1}{A}. \tag{6}$$

The force F_1 acting on the particle can be expressed by the so-called Newton formula:

$$F_1 = \frac{\varrho_g}{2} A_0 C_e w^2. \tag{7}$$

The drag coefficient C_e depends on the shape of the particle, its surface roughness, and the Reynolds' number Re_0 of the stream around it:

$$Re_0 = d_0 w / \nu_g. \tag{8}$$

3.2 Additional Pressure Drop of Pneumatic Conveying

The additional pressure drop of pneumatic conveying will be examined under the following conditions:

a) The expansion of the gas will be neglected (we calculate with $\varrho_g =$ const in spite of the pressure drop).

b) The dimension of the grains in the investigation will be small compared to diameter D of pipe so that the drag coefficient C_e does not change considerably compared to its infinite space value.

c) Particles are found so rarely in the pipe that there is no interaction between them. (This is true for thin flow conveying.)

d) The change of the drag coefficient C_e with Re_0 will be neglected.

The above conditions are fulfilled with good approximation in thin flow conveying of granular matter ($d_0 > 0.1$ mm).

3.2.1 Velocities and Pressures during Settling

A settling velocity w_0 for testing equipment will first be examined. The settling state is a boundary situation of the pneumatic conveying (*Fig. 9*).

The velocity of the relatively quiescent settling particles may be taken for $v_s = 0$. Relative velocity is then equal to the gas velocity itself: $v_g = w_0$ which is called the settling velocity. With this condition the aerodynamic force acting on each grain of mass m_1 and weight $G_1 = m_1 g$ is:

$$G_1 = F_1.$$

The additional pressure drop in a pipe of length ℓ is the lifting pressure drop Δp_e:

$$\Delta p_j = \frac{\Sigma F_1}{A} = \frac{\Sigma G_1}{A} = \Delta p_e,$$

because $F_1 = G_1$. Now the additional pressure drop may be called lifting pressure drop because it compensates the gravitational force.

From the force equilibium $G_1 = F_1$ formed during the settling the settling velocity w_0 can also be determined:

$$w_0 = \sqrt{\frac{2 m_1 g}{\varrho_g A_0 C_e}} = \sqrt{\frac{4 \varrho_s g}{3 \varrho_g C_e} d_0}. \tag{7/a}$$

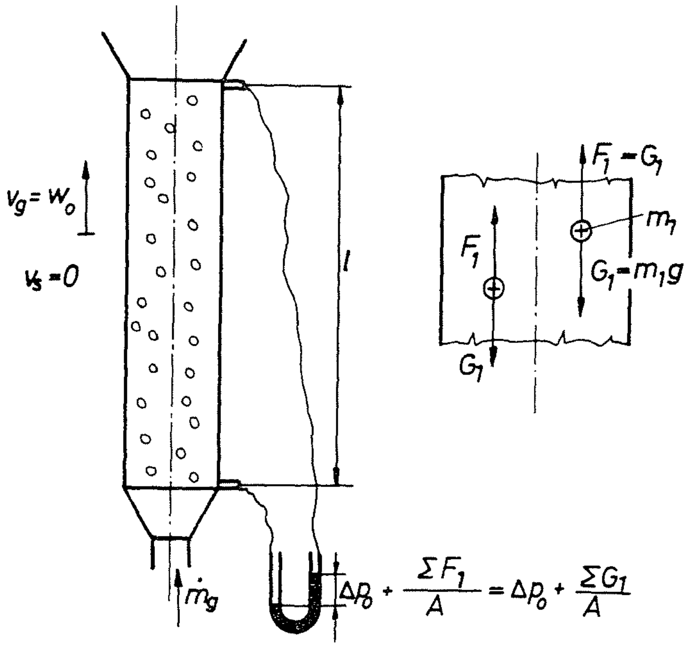

Fig. 9. Forces and velocities during settling

3.2.2 Stationary Pneumatic Conveying

Fig. 10 shows forces and velocities arising in stationary conveying. In pneumatic conveying velocities can be interpreted as follows:

The gas velocity is:

$$v_g = \frac{Q_g}{A} = \frac{\dot{m}_g}{\varrho_g A}, \tag{9}$$

which means an average for cross-section calculated from the amount of flowing gas.

The material velocity is:

$$v_s = \frac{\dot{m}_s}{q_s} \tag{10}$$

which means an average in axial direction in the pipe. It can be determined with measurements from the amount of matter q_s staying at a given time in a pipe length of 1 m.

Examining the motion of a single particle it is seen that it does not move on a straight way but colliding with the wall due to the turbulent

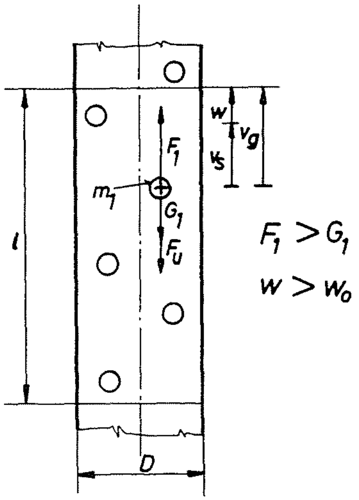

Fig. 10. Stationary conveying. Forces and velocities

flowing of the gas moving on an inclined path. Due to collision, a part of its velocity will be lost and afterwards the gas will accelerate it until the next collision. According to the interpretation of v_s, the particle moving with this average velocity would reach for t time the same point as with its actual, varying velocity.

As to dynamics, collision to the pipe wall causes that all particles with mass m_1 are hindered in their motion by a force F_u that can be supposed to be continuous. On a path of length ℓ, this so-called collision force [10] is consuming a work equal to the decrease of kinetic energy:

$$F_u l = \frac{\ell}{D}\xi\frac{m_1 v_s^2}{2} = k_u \frac{\ell}{D} m_1 v_s^2. \tag{11}$$

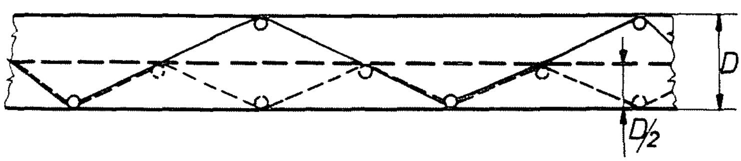

Fig. 11. The number of collisions is inversely proportional to the pipe diameter D during thin flow conveying

Here $\xi = 2k_u$ is a coefficient indicating the reduction of kinetic energy due to collisions in a pipe of 1 m length. Kinetic energy reduction is inversely proportional to the pipe diameter D because the grains are supposed to move straightly with a small angle to the axis of the pipe and thus the frequency of collision is inversely proportional to the pipe diameter (*Fig 11*). From (11) the collision force:

$$F_u = k_u \frac{m_1 v_s^2}{D}. \tag{11/a}$$

Here k_u is the collision factor, depending on the material of the grain and of the wall. The particle moving with v_s average velocity loses $2k_u$ part of its kinetic energy due to the collision with the wall of the pipe of D diameter. This energy loss will be compensated by the so-called collision pressure drop Δp_u which is necessary to the reacceleration of the n of particles in the pipe of D diameter:

$$n = l \frac{\dot{m}_s}{v_s m_1} \tag{12}$$

and:

$$\Delta p_u = \frac{n F_u}{A} = k_u \frac{\ell}{D} \frac{\dot{m}_s v_s}{A}. \tag{13}$$

The force F_1 moving the particle ensures uniform velocity by compensating the gravitational force G_1 and the continuous collision force F_u:

$$F_1 = G_1 + F_u. \tag{14}$$

Due to this force equilibrium, in the case of vertical conveying the w relative velocity can only be greater than the velocity of fall:

$$w > w_0, \tag{15}$$

because a motive force greater than the gravitational force ($F_1 > G_1$) can only be produced at a relative velocity greater than the velocity of fall.

In the case of vertical conveying, due to the superposition of pressures, the additional pressure drop Δp_j of conveying consists of two parts which are the lifting pressure drop Δp_e necessary to balance weight and the collision pressure drop (Δp_u):

$$\Delta p_j = \Delta p_e + \Delta p_u, \tag{16}$$

the lifting pressure drop:

$$\Delta p_e = \frac{n G_1}{A} = k_\ell l \frac{\dot{m}_s g}{v_s A}, \tag{17}$$

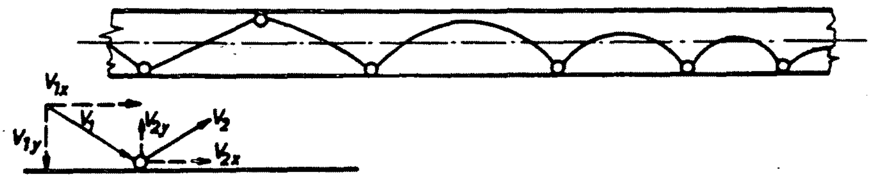

Fig. 12. The moving of the grain without aerodynamical force

where k_e is the lifting factor. Its value is $k_e = 1$ for vertical conveying, $k_e < 1$ for conveying of upwards slope and for horizontal conveying.

The value of k_e is not equal to zero in horizontal conveying either, because the effect of velocity change produced by the collision (see *Fig. 12*) must be recompensated. This is the condition for the conservation of the average velocity, that is, for steady motion. The axial component of the velocity (v_{1x} in *Fig. 12*) of the particle coming decreases to v_{2x} due to collision. This can be restored by the Δp_u collision pressure drop providing reacceleration for the n pieces of particles in the pipe of length ℓ. The reduction of the velocity component from v_{1y} to v_{2y} perpendicular to the pipe axis will be recompensated by the lifting pressure drop Δp_e.

If the aerodynamic force moving the particle ceased (e.g. particle moving in vacuum was studied), the particle would deviate from its original state of motion due to collision. It would arrive at the bottom of the pipe with an axial deceleration (*Fig. 12*). To avoid this effect, the grain must also be lifted in a horizontal pipe.

Thus the additional pressure drop of the thin flow conveying for vertical, declined and horizontal pipe equally is:

$$\Delta p_j = \Delta p_u + \Delta p_e \tag{18}$$

and

$$\Delta p_u = k_u \frac{\ell}{D} \frac{\dot{m}_s v_s}{A}; \tag{13}$$

$$\Delta p_e = k_e l \frac{\dot{m}_s g}{v_s A}, \tag{17}$$

where k_u collision and the k_e lifting coefficients depend partly on the elastic properties of the material conveyed and the pipe and on the pipe's angle to horizontal.

For the calculation of the components Δp_u and Δp_e of the additional pressure drop Δp_j the material velocity v_s must be known. This can be determined from the equilibrium of the forces acting on the grains:

$$F_1 = k_e G_1 + F_u, \tag{19}$$

that is

$$\frac{\varrho_g}{2} A_0 C_e w^2 = k_e m_1 g + k_u \frac{m_1 v_s^2}{D} \qquad (19/a)$$

where $w = v_g - v_s$ and with the substitution

$$m_1 = \frac{G_1}{g} = \frac{\varrho_g}{2g} A_0 C_e w_0^2$$

$$v_g = v_s + w_0 \sqrt{k_e + k_u v_s^2 / gD} \qquad (20)$$

the relation between gas velocity and material velocity can be obtained.

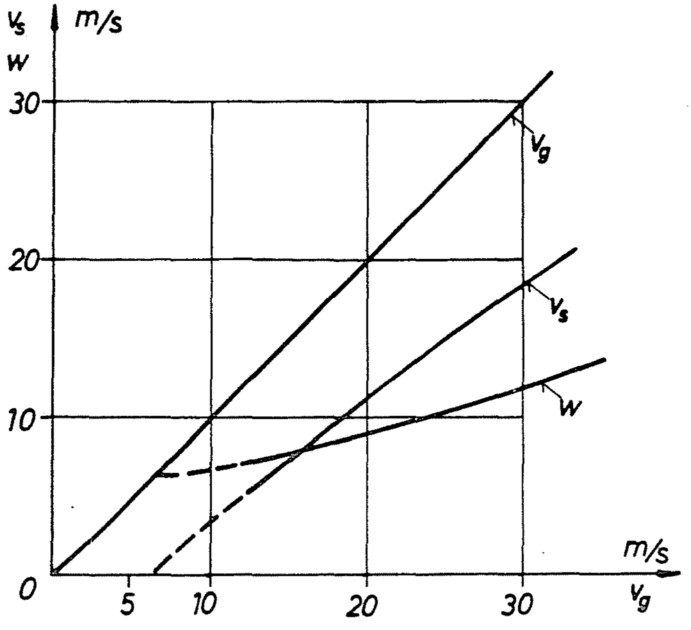

Fig. 13. Material velocity v_s and relative velocity w as a function of gas velocity v_g during vertical stationary conveying of sand. $\varrho_s = 2420$ kg/m^3, $d_0 = 1$ mm, $\varrho_g = 1.23$ kg/m^3, $w_0 = 6.7$ m/s, $k_e = 1$, $k_u = 0.0035$, $D = 60$ mm

Fig. 13 shows velocities developing in the vertical, uniform conveying of sand, calculated from (20). Material parameter data (w_0, d_0, ϱ_s, etc.) are from WEBER [7] [p. 123]

EXAMPLE 1:

Sand ($d_0 = 1$ mm, $\varrho_s = 2420$ kg/m^3, $w_0 = 6.7$ m/s) is being vertically conveyed with $\dot{m}_s = 3$ t/h $= 0.83$ kg/s in a pipe of diameter $D = 60$ mm, cross-section $A = 0.00283$ m^2 with air of $v_g = 24$ m/s velocity ($\varrho_g =$ 1.23 kg/m^3).

The mass flow of the conveying air is :

$$\dot{m}_g = A v_g \varrho_g = 0.083 \text{ kg/s}.$$

The mass ratio of feeding is: $\mu = \frac{\dot{m}_\ell}{\dot{m}_g} = 10$.

With coefficients $k_e = 1$ and $k_u = 0.0035$ from *Fig. 13* the material velocity in the uniform velocity section is $v_s = 14$ m/s, the relative velocity is $w = 10$ m/s. The relative lag of the material (slip) is: $s = w/v_g = 0.417 = 41.7$ %.

In the steady section the ratio of masses staying in the pipe is in the same time:

$$q_s/q_g = \frac{\mu}{1-s} = 17.2.$$

The cross-section taken by the material is:

$$A_s = \frac{\dot{m}_s}{v_s \varrho_s} = 0.0000245 \text{ m}^2$$

which is $A_s/A = 0.0087 = 0.87$ % of the pipe cross-section

The narrowing is smaller than 1% thus it is a thin flow conveying.

Far from the inlet, in an $\ell_{st} = 15$ m long section of uniform velocity in no-load state with $\lambda = 0.02$ pipe friction coefficient the pressure drop is:

$$\Delta p_0 = \lambda \frac{\ell_{st}}{D} \frac{v_g^2}{2} \varrho_g = 1770 \text{ Pa}.$$

The pressure drop necessary to lift the weight of the material:

$$\Delta p_e = k_e \ell_{st} \frac{\dot{m}_s g}{v_s A} = 3083 \text{ Pa}.$$

Collision pressure drop needed to reaccelerate the particles after colliding with the wall:

$$\Delta p_u = k_u \frac{\ell_{st}}{D} \frac{\dot{m}_s v_s}{A} = 3592 \text{ Pa},$$

thus the additional pressure drop:

$$\Delta p_j = \Delta p_e + \Delta p_u = 6675 \text{ Pa}.$$

Total pressure drop:

$$\Delta p = \Delta p_0 + \Delta p_j = 8445 \text{ Pa}.$$

3.3 Additional Pressure Drop in the Starting Section

In the starting section beside the collision and lifting pressure drops considerable pressure difference is also necessary for the acceleration of the solid particles. Furthermore, an additional lifting pressure drop exceeding that of the steady section must also be considered in the vertical starting section.

3.3.1 The Accelerating Pressure Drop

Fig. 3 also shows velocities and pressures in the starting section of a horizontal pipe. The solid material of mass flow $\dot{m}_s$ is accelerating from $v_s = 0$ to $v_{s\infty}$ velocity. Examining the flowing of the solid matter the accelerating pressure drop Δp_d can be calculated from the theorem of the momentum:

$$\dot{m}_s \Delta v_s = \Delta p_d A.$$

When accelerating from $v_s = 0$ to $v_{s\infty}$, thus the accelerating pressure drop is:

$$\Delta p_d = \frac{\dot{m}_s v_{s\infty}}{A}. \tag{21}$$

Naturally the same result will be obtained calculating from the elemental length $\mathrm{d}\ell$ of the starting section, from the force $\mathrm{d}F_d$ necessary to accelerate with a_s the mass $\mathrm{d}m_s$ of the matter and from the pressure producing this force:

$$\mathrm{d}F_d = a_s \mathrm{d}m_s = A\mathrm{d}p_d.$$

The necessary accelerating pressure drop is the integral of the elemental accelerating pressures $\mathrm{d}p_d$ on the starting section:

$$\Delta p_d = \int \mathrm{d}p_d = \frac{1}{A}\int \mathrm{d}F_d = \frac{1}{A}\int a_s \mathrm{d}m_s$$

with the substitutions $a_s = \frac{\mathrm{d}v_s}{\mathrm{d}t}$; $\mathrm{d}m_s = q_s \mathrm{d}l = \frac{\dot{m}_s}{v_s}\mathrm{d}\ell$; and $v_s = \frac{\mathrm{d}\ell}{\mathrm{d}t}$

$$\Delta p_d = \frac{\dot{m}_s}{A}\int_0^{v_{s\infty}} dv_s = \dot{m}_s v_{s\infty}/A.$$

The obtained result is equal to (21).

According to (21), the pressure drop needed for the acceleration of the solid matter is independent of the direction of the conveying pipe (horizontal, vertical or inclined). For calculating it the steady operation velocity of the matter $v_{s\infty}$ must be known. $v_{s\infty}$ can be obtained from *Eq.* (20).

The *Eq.* (21) for the accelerating pressure drop is also formally similar to *Eq.* (13) for the collision pressure drop the latter reaccelerating the particles after the axial loss of velocity. Thus the double of the collision coefficient $2k_u = \xi$ may be interpreted as the loss of kinetic energy in a pipe of $D = 1$ m diameter and $\ell = 1$ m length. It is seen in example 1 from the value $2k_u = \xi = 0.007$ for sand that in a pipe of $D = 0.06$ m diameter $\xi/D = 0.117$/m, which means that the solid particles lose 11.7% of their kinetic energy on each meter due to collisions with the wall. Δp_u will compensate this energy loss by reacceleration.

It must be emphasized that the pressure drop Δp_d needed for accelerating the solid material is considerably greater than the theoretical pressure drop Δp_{d0} that would be needed for increasing the kinetic energy. The power of the kinetic energy increase:

$$P_{d0} = \frac{\dot{m}_s v_{s\infty}^2}{2}.$$

Could the accelerating gas transfer this to the solid phase without loss, Δp_{d0} pressure drop would only be formed:

$$\frac{\dot{m}_s v_{s\infty}^2}{2} = Q_g \Delta p_{d0}.$$

Then:

$$\Delta p_{d0} = \frac{\dot{m}_s v_{s\infty}^2}{2Q_g} = \frac{1}{2}\frac{v_{s\infty}}{v_g}\frac{\dot{m}_s v_{s\infty}}{A},$$

that is,

$$\Delta p_{d0} = \frac{1}{2}(1 - s)\Delta p_d,$$

where s is the slip, the relative lag of the solid matter behind the gas.

Hence the value of the accelerating pressure drop developing is more than twice of the supposed one on the base of energetical considerations. The reason is that the velocity of the material is smaller than the gas velocity. (This phenomenon resembles the considerable energy loss due to the slip of the mechanical coupling when starting a motor-car. It is to be noted that the gas and solid phase velocity difference during steady operation also continuously causes a so-called slip loss, similarly to the car going with slipping coupling even at constant velocity.)

EXAMPLE 2: The accelerating drop from example 1 on sand conveying will be determined.

Data:
Mass flow of the conveyed matter: $\dot{m}_s = 0.83$ kg/s,
Conveying pipe diameter: $D = 60$ mm, cross-section $A = 0.00283$ m^2,
Velocity of gas: $v_g = 24$ m/s,
Material velocity: $v_{s\infty} = 14$ m/s,
Accelerating pressure drop:

$$\Delta p_d = \frac{\dot{m}_s v_{s\infty}}{A} = 4106 \text{ Pa}.$$

The significance of the accelerating pressure drop can be valued when comparing it with the additional pressure drop of the uniform conveying. It can be calculated from the data of example 1 that the accelerating pressure drop is identical with the additional pressure drop in a steady section of 9.2 m length.

Beside this, as it has already been mentioned, in the vertical starting section another increase of the pressure drop due to lifting of weight must be calculated with.

3.3.2. Weight-Lifting Pressure Drop in the Vertical Starting Section

It has been displayed in connection with *Fig. 6* that there is more material staying in the vertical starting section ℓ_{inst} at the same time than in a section of uniform conveying of the same length. The varying values of mass per running meter q_s are also seen in the figure. Lifting pressure drop in the starting section cannot be calculated with the (17) formula of the steady operation as material velocity is lower in the entire starting section than in the steady section ($v_s < v_{s\infty}$). Here the quotient of the weight of the material staying in the starting section ℓ_{inst} and the pipe cross-section must be determined.

Lifting pressure drop in the starting section:

$$\Delta p_{ei} = \frac{g}{A}\int_0^{\ell_{\text{inst}}} \mathrm{d}m_s = \frac{g}{A}\int_0^{\ell_{\text{inst}}} q_s dl = \frac{g\dot{m}_s}{A}\int_0^{\ell_{\text{inst}}} \frac{1}{v_s}\mathrm{d}\ell =$$

$$= \frac{g\dot{m}_s}{A}\int_0^{t_i} \mathrm{d}t = \frac{g\dot{m}_s}{A} t_i, \qquad (22)$$

where t_i is the dwelling time of the grains of solid in the starting section.

Hence for the determination of the lifting pressure drop Δp_{ei} in the starting section either the changing of the material velocity v_s along the pipe length or the residing time t_i must be known.

Therefore the equation of motion of the solid phase in the starting section will be determined:

$$F_1 - k_e G_1 - F_u = m_1 \frac{dv_s}{dt}.$$

After substitution (examining vertical conveying, $k_e = 1$)

$$\frac{\varrho_g}{2} A_0 C_e (v_g - v_s)^2 - k_e \frac{\varrho_g}{2} A_0 C_e w_0^2 - k_u \frac{m_1 v_s^2}{D} = m_1 \frac{dv_s}{dt}.$$

Substituting $m_1 = G_1/g$ into this differential equation, the solution is

$$v_s = v_g \beta \frac{1 - e^{-\alpha t}}{1 - \delta e^{-\alpha t}} = v_{s\infty} \frac{1 - e^{-\alpha t}}{1 - \delta e^{-\alpha t}}. \tag{23}$$

From the inverse of this, the starting time t_i can also be calculated with a given approximation of the end velocity (e.g. $v_s/v_{s\infty} = 0.95$):

$$t_i = -\frac{1}{\alpha} \ln \frac{1 - v_s/v_{s\infty}}{1 - \delta v_s/v_{s\infty}}. \tag{24}$$

In technical practice starting is considered to be finished when the velocity has reached its end value ($v_{s\infty}$, material velocity in steady-state) with 5% deviation that is, $v_s/v_{s\infty} = 0.95$.

By knowing the $v_s(t)$ function (23) the length of the starting path can also be determined:

$$\ell_{\text{inst}} = \int_0^{t_i} v_s \mathrm{d}t,$$

that is,

$$\ell_{\text{inst}} = v_{s\infty} \left(t_i - \frac{1 - \delta}{\alpha \delta} \ln \frac{1 - \delta e^{-\alpha t_i}}{1 - \delta} \right) \tag{25}$$

the data in (23), (24) and (25):

$$\alpha = \frac{2g}{w_0} B; \qquad \delta = \frac{v_g - w_0 B}{v_g + w_0 B};$$

and

$$B = \sqrt{k_e + k_u \frac{v_g^2 - k_e w_0^2}{gD}}.$$

$v_{s\infty}$ can be determined from *Eq.* (20) or

$$v_{s\infty} = \beta v_g,$$

where

$$\beta = \frac{1-(w_0/v_g)^2}{1+Bw_0/v_g}.$$

From the above equations, the lifting pressure drop in the starting section Δp_{ei} can be calculated by determining t_i and deviation from the lifting pressure drop of the uniform conveying can be calculated by determining ℓ_{inst}.

EXAMPLE 3: Continuing example 2, lifting pressure drop in a vertical starting section will be determined.

The data of sand conveying: sand with $\dot{m}_s = 0.83$ kg/s will be vertically conveyed with $v_g = 24$ m/s air velocity in a pipe of $D = 60$ mm diameter and $A = 0.00283\ \text{m}^2$ cross-section. Material velocity in steady operation: $v_{s\infty} = 14$ m/s, the settling velocity $w_0 = 6.7$ m/s, $k_e = 1$ (vertical conveying), $k_u = 0.0035$.

The data from *Eqs.* (23), (24) and (25):

$$B = \sqrt{k_e + k_u(v_g^2 - k_e w_0^2)/gD} = 2.039,$$
$$\alpha = 2gB/w_0 = 5.97s^{-1},$$
$$\delta = (v_g - w_0 B)/(v_g + w_0 B) = 0.2745.$$

With these, the time function of the material velocity is:

$$v_s = 14(1 - e^{5.97t})/(1 - 0.2745e^{-5.97t})$$

Its values are seen in *Fig. 14*.

The starting time (which means the dwelling time of the individual sand particles in the starting section) is at the value of $v_s/v_{s\infty} = 0.95$ (that is, the stationary velocity is approximated to 5 %):

$$t_i = -\frac{1}{\alpha}\ln\frac{1 - v_s/v_{s\infty}}{1 - \delta v_s/v_{s\infty}} = 0.451\ \text{s}$$

and the length of the starting path (starting section):

$$\ell_{\text{inst}} = v_{s\infty}\left(t_i - \frac{1-\delta}{\alpha\delta}\ln\frac{1-\delta e^{-\alpha t_i}}{1-\delta}\right) = 4.44\ \text{m}.$$

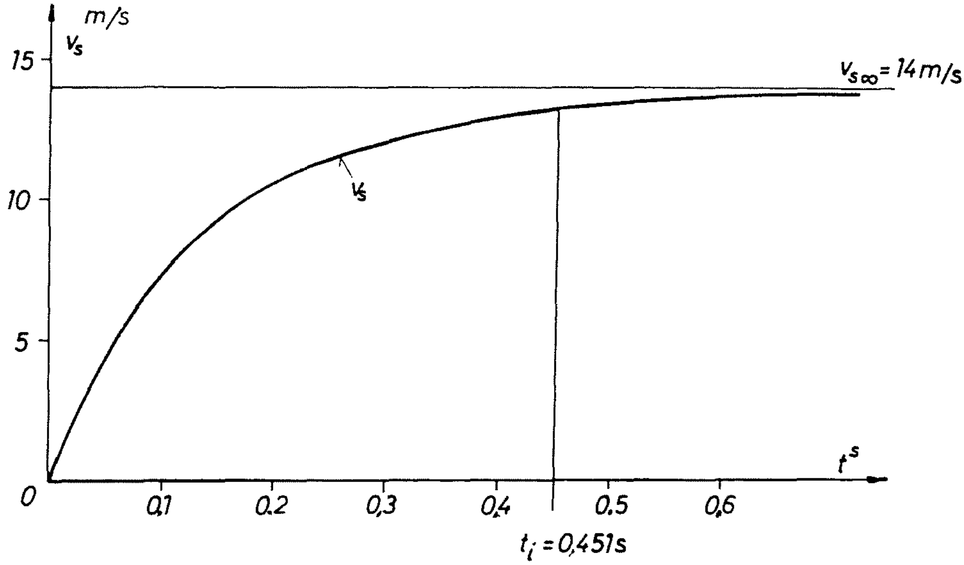

Fig. 14. Velocity of sand as a function of time in the starting section, $v_g = 24$ m/s, $D = 60$ mm

The lifting pressure drop in the starting section:

$$\Delta p_{ei} = \frac{g\dot{m}_s}{A} t_i = 1298 \text{ Pa}.$$

This considerably exceeds the lifting pressure drop of a pipe of the steady operation mode and of the same length as the starting section.

The lifting pressure drop is per meter in the steady section with the data of example 1:

$$\frac{\Delta p_e}{\Delta \ell_{st}} = 205.5 \text{ Pa/m}.$$

The lifting pressure drop is in a pipe section of the same length as the starting section ($\ell_{\text{inst}} = 4.44$ m) and is in steady operation mode:

$$\ell_{\text{inst}} \frac{\Delta p_e}{\ell_{st}} = 912.5 \text{ Pa}.$$

Additional pressure due to lifting in the starting section:

$$\Delta p_e^* = \Delta p_{ei} - \ell_{\text{inst}} \frac{\Delta p_e}{\ell_{st}} = 385 \text{ Pa}.$$

Additional pressure compared to uniform conveying in the vertical starting section (from example 2, $\Delta p_d = 4106$ Pa):

$$\Delta p_i = \Delta p_d + \Delta p_e^* = 4491 \text{ Pa}.$$

This exceeds the accelerating pressure drop by about 10% that is also verified by the measurements of FLATOW (*Fig. 7*) [6].

Consequently, the theoretical investigations and the measurements show that pressure drops and velocities in thin flow pneumatic conveying can properly be calculated by the method based on the forces acting on the particles.

Nomenclature

$A = D^2\Pi/4$	(m^2)	pipe cross-section
A_0	(m^2)	grain cross-section
C_e	—	drag coefficient
D	(m)	pipe diameter
d_0	(m)	grain diameter
F_1	(N)	aerodynamic force acting on a grain
F_u	(N)	collision force
G_1	(N)	weight of one grain
g	(m/s^2)	gravitational acceleration
k_e	—	lifting coefficient
k_u	—	collision coefficient
ℓ	(m)	pipe length
ℓ_{inst}	(m)	length of starting section
ℓ_{st}	(m)	pipe length of stationary section
m_s	(kg)	mass of solid matter
$\dot{m}_g$	(kg/s)	mass flow of gas
$\dot{m}_s$	(kg/s)	mass flow of solid
m_1	(kg)	mass of one grain
p	(Pa)	pressure
Δp	(Pa)	differential pressure
Δp_b	(Pa)	inlet pressure drop
Δp_d	(Pa)	accelerating pressure drop
Δp_e	(Pa)	lifting pressure drop
Δp_i	(Pa)	additional pressure drop in the starting section
Δp_j	(Pa)	additional pressure drop
Δp_{p0}	(Pa)	no-load ($\dot{m}_s = 0$) pressure drop of the pipe
Δp_0	(Pa)	no-load ($\dot{m}_s = 0$) pressure drop
Δp_u	(Pa)	collision pressure drop

Q_g	(m^3/s)	volume flow of gas
$q_g = \dot{m}_g/v_g$	(kg/m)	mass of gas per metre
$q_s = \dot{m}_s/v_s$	(kg/m)	mass of solid per metre
$Re_0 = \frac{d_0 w}{\nu_g}$	—	Reynolds' number
$s = w/v_g$	—	slip, relative lag
t	(s)	time
t_i	(s)	dwelling time in the starting section
v_g	(m/s)	gas velocity
v_s	(m/s)	material velocity
$v_{s\infty}$	(m/s)	material velocity in stationary conveying
$w = v_g - v_s$	(m/s)	relative velocity
w_0	(m/s)	settling velocity
ζ_b	—	inlet loss coefficient
λ	—	pipe friction coefficient
$\mu = \dot{m}_s/\dot{m}_g$	—	feeding mass ratio
ν_g	(m^2/s)	kinematic viscosity
ϱ_g	(kg/m^3)	gas density
ϱ_s	(kg/m^3)	material density

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