

MASS TRANSFER BIOT NUMBERS

Mihály PARTI

Department of Chemical and Food Engineering
Technical University of Budapest
Budapest, Hungary

Received: Febr. 8, 1994

Abstract

The heat transfer Biot number has a key role in studying transient heat transfer. In principle an analogous mass transfer Biot number can be derived for transient mass transfer. Substituting that mass transfer Biot number for the heat transfer Biot number all the results and statements of transient heat transfer are valid for transient mass transfer. In our experience the mass transfer Biot number used in the relevant literature for that aim does not give similarity between the two transfer processes. Using a simple model it was shown that three mass transfer Biot numbers can be derived: the fluid-phase, the solid-phase and the analogous mass transfer Biot number. In the relevant literature the fluid-phase mass transfer Biot number is predominantly used which does not give similarity between the two processes. The solid-phase mass transfer Biot number gives a limited similarity of the two processes because of the limited boundary condition used at the interface. The mass transfer Biot number based on the equilibrium conception at the interface gives a general similarity. This assumes only equilibrium at the interface for both processes, consequently it handles the two transfer processes on the same basis.

Keywords: similarity, heat transfer Biot number, fluid-phase, solid-phase and analogous mass transfer Biot number.

Introduction

The fundamental role of the heat transfer Biot number in transient heat transfer is well known. It shows the ratio of the conductive resistance within a body to the convective resistance outside the body. Consequently, it provides a measure of the temperature drop inside the body to the temperature difference between the body surface and the bulk fluid. Hence, when confronted with such a problem, the very first thing one should do is to calculate the Biot number. If the condition $Bi_H < 0.1$ is satisfied the lumped parameter model can be used since the conductive resistance to heat transfer can be neglected relative to the convective one. If $Bi_H > 0.1$ a more complicated distributed parameter model should be used (SCHLÜNDER, 1983). There is a strong preference for using the lumped parameter model since it is the simplest and most convenient method that can be used to solve con-

duction problems. Hence it is important to determine under which conditions it may be used with reasonable accuracy.

According to the analogy between the transport processes an analogous Biot number for transient mass transfer can be derived. That mass transfer Biot number should have the same role in mass transfer that the heat transfer Biot number has in heat transfer. By its application all the statements and results including that in the preceding paragraph of transient heat transfer can be utilized for transient mass transfer substituting the heat transfer Biot number by the analogous mass transfer Biot number.

However, we have found that the proper mass transfer Biot number is not well clarified and an erroneous one is generally used.

The objectives of the present study are:

- to derive the various mass transfer Biot numbers by using a simple model;
- to choose the mass transfer Biot number analogous to the heat transfer Biot number;
- to show the application.

The Model

Consider a slab of thickness $2L$ at the left side of *Fig. 1*, with sealed edges on four sides, so that heat transfer can take place only toward and from the flat parallel faces. Suppose that it is initially at a uniform temperature T_0 and experiences convection cooling at its surfaces when immersed in a fluid of $t_b < T_0$. The temperature distribution at any time is shown by the figure. A similar plane at the right side of *Fig. 1* is shown which is initially at a uniform concentration C_{A0} and undergoes isothermal convection mass transfer at its surfaces when submerged in a fluid of c_{Ab} . The concentration distribution at any time is also shown on the figure. Subscript A refers to the diffusing component. The conditions at the interface are also shown by the figure.

The final temperature of the solid is the bulk fluid temperature, that is $T_f = t_b$, when *temperature equilibrium* exists between the solid and the fluid. Subsequently, t_b will be called the *equilibrium temperature* and substituted by T_e :

$$T_f = t_b = T_e . \quad (1)$$

After Lewis and Whitman no mass transfer resistance is assumed across the interface separating the solid and the fluid (TREYBAL, 1968). As a result the concentrations at the interface, C_{As} and c_{As} , are equilibrium values given by the isothermal equilibrium curve of the system:

$$C_{As} = \varphi(c_{As}) . \quad (2)$$

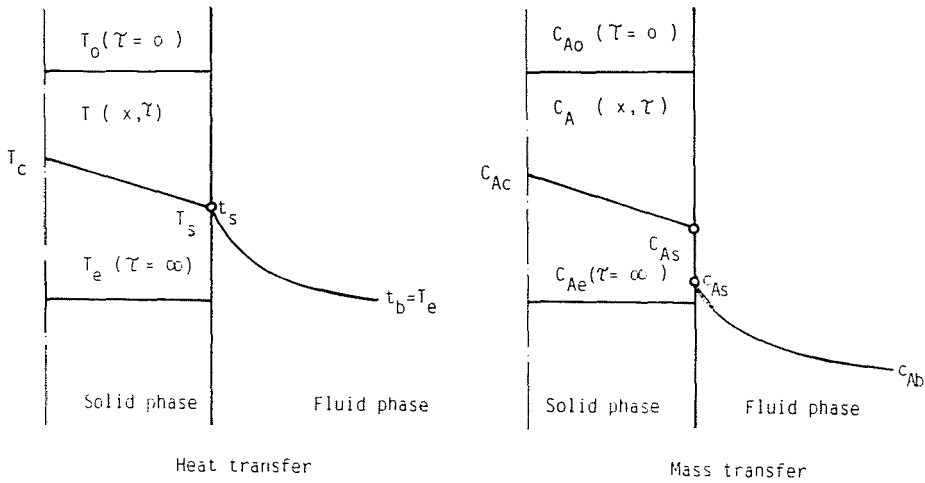


Fig. 1. The model

This assumption applied to mass transfer is in accordance with the one accepted generally for heat transfer assuming that the fluid and the solid temperatures at the interface are equal if there is no contact resistance at the interface, that is $t_s = T_s$. Of course, we can also say for heat transfer that there is equilibrium at the interface and the temperatures are equilibrium values which means their agreement without contact resistance.

The final concentration of the solid is in equilibrium with the main body of the fluid and described by:

$$C_{Af} = C_{Ae} = \varphi(c_{Ab}) . \tag{3}$$

C_{Ae} is called the *equilibrium concentration*.

The difference between the two transport processes can be clearly seen: the equilibrium temperature T_e is equal to the fluid bulk temperature t_b , conversely the equilibrium concentration C_{Ae} differs from the fluid bulk concentration c_{Ab} . This difference results in difference between the heat and mass transfer Biot numbers and possibility of the definition of different mass transfer Biot numbers.

The problems depicted may be treated as one dimensional in x . We are interested in the temperature as well as the concentration variation with position and time inside the plane, $T(x, t)$ and $C_A(x, t)$. With no internal generation and the assumption of constant thermal as well as mass conductivity, the heat and the mass diffusion equations reduce to:

$$\begin{array}{ll} \text{for heat transfer} & \text{for mass transfer} \\ \frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial x^2}, & \frac{\partial C_A}{\partial \tau} = D \frac{\partial^2 C_A}{\partial x^2}. \end{array} \quad (4)$$

To solve the Eq. (4) for the temperature and the concentration distribution $T(x, \tau)$ and $C_A(x, \tau)$ the initial and the boundary conditions should be specified:

the initial conditions are:

$$T(x, 0) = T_o \quad C_A(x, 0) = C_{Ao} \quad (5)$$

and the boundary condition are:

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C_A}{\partial x} \right|_{x=0} = 0. \quad (6)$$

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=L} = h(t_s - t_b), \quad -D \left. \frac{\partial C_A}{\partial x} \right|_{x=L} = k_c(c_{As} - c_{Ab}). \quad (7)$$

The assumptions applied were the following: the temperature and concentration distribution initially are uniform and symmetrical during the process and convective transport takes place at the interface.

To generalize the results dimensionless terms are introduced, namely the dimensionless temperature and the dimensionless concentration:

$$\vartheta = \frac{T - T_e}{T_o - T_e}, \quad \nu = \frac{C_A - C_{Ae}}{C_{Ao} - C_{Ae}}, \quad (8)$$

the dimensionless spatial coordinate:

$$\eta = \frac{x}{L} \quad (9)$$

and the Fourier number for heat transfer and for mass transfer:

$$Fo_H = \frac{\alpha \tau}{L^2}, \quad Fo_M = \frac{D \tau}{L^2}. \quad (10)$$

Substituting the dimensionless terms of Eqs. (8) to (10) into Eqs. (4) to (7), the model becomes:

for heat transfer

$$\frac{\partial \vartheta}{\partial Fo_H} = \frac{\partial^2 \vartheta}{\partial \eta^2},$$

$$\vartheta(\eta, 0) = 1,$$

$$\left. \frac{\partial \vartheta}{\partial \eta} \right|_{\eta=0} = 0,$$

for mass transfer

$$\frac{\partial \nu}{\partial Fo_M} = \frac{\partial^2 \nu}{\partial \eta^2}, \tag{11}$$

$$\nu(\eta, 0) = 1, \tag{12}$$

$$\left. \frac{\partial \nu}{\partial \eta} \right|_{\eta=0} = 0 \tag{13}$$

and Eq. (7) is separated into Eq. (14) on heat transfer:

$$-\left. \frac{\partial \vartheta}{\partial \eta} \right|_{\eta=1} = \frac{hL}{k} \vartheta(1, Fo_H) \tag{14}$$

and into Eq. (15) on mass transfer:

$$-\left. \frac{\partial \nu}{\partial \eta} \right|_{\eta=1} = \frac{k_c L}{D} \frac{c_{As} - c_{Ab}}{C_{Ao} - C_{Ae}}. \tag{15}$$

In Eq. (14) the heat transfer Biot number Bi_H is:

$$Bi_H = \frac{hL}{k} \tag{16}$$

which using Eqs. (1) and (7) as well as Fig. 1 can be described in the following form:

$$Bi_H = \frac{T_c - T_s}{t_s - t_b} = \frac{T_c - T_s}{T_s - T_e}. \tag{17}$$

The solution of the heat transfer model using the dimensionless terms is the following:

$$\vartheta = \varphi_H(\eta, Fo_h, Bi_H) \tag{18}$$

and the average temperature:

$$\bar{\vartheta} = \varphi_H(Fo_H, Bi_H). \tag{19}$$

The solution of the mass transfer model using the dimensionless terms is the following:

$$\nu = \varphi_M(\vartheta, Fo_M, Bi_M) \tag{20}$$

and the average concentration

$$\bar{\nu} = \varphi_M(Fo_M, Bi_M). \tag{21}$$

The analogous mass transfer Biot number Bi_M was introduced in the latest equation in advance.

It is known that the following requirements have to be met for the similarity (e.g. LANGHAAR, 1951): the two systems, that is, the heat transfer system and the mass transfer system or two mass transfer systems, must

- be geometrically similar;
- have identical initial and boundary conditions and
- have identical equations.

These requirements have to be taken into account for the derivation of the mass transfer Biot number. The geometrical similarity is assumed automatically. Next the similarity of the mathematical models has to be achieved. If these requirements are met all the statements and solutions on heat transfer can be utilized for mass transfer by merely substituting the variables in *Eqs.* (18) and (19) or the results on one mass transfer system in *Eqs.* (20) and (21) can be utilized on a second mass transfer system.

Mass Transfer Biot Numbers

Fluid-Phase Mass Transfer Biot Number

The ratio appearing in *Eq.* (15):

$$Bi_{Mf} = \frac{k_c L}{D} \quad (22)$$

can be considered as the fluid-phase mass transfer Biot number. This is formally similar to the heat transfer Biot number since the convective heat transfer coefficient h and the thermal conductivity k are substituted by the convective mass transfer coefficient k_c and the mass conductivity D , respectively. This formal similarity may be the explanation that Bi_{Mf} is commonly considered the mass transfer Biot number being analogous to Bi_H and having the same role for mass transfer as the heat transfer Biot number has for heat transfer. Namely, it is assumed that when Bi_{Mf} is less than 0.1 the internal resistance to mass transfer can be neglected and the mass transfer process can be described by a lumped parameter model (BRUIN and LUYBEN, 1980; IMRE and NÉMETH, 1974; KEEY, 1972; MEIERING, HOFFMANN and BAKKER-ARKEMA, 1970; PAKOWSKI and MUJUMDAR, 1987).

However, *Eq.* (15) is not analogous to *Eq.* (14) since the ratio of the concentration differences in the fluid and in the solid is not equal to the

dimensionless concentration of Eq. (8) at the interface:

$$\frac{c_{As} - c_{Ab}}{c_{Ao} - c_{Ae}} \neq \nu(1, F_{OM}) = \frac{C_{As} - C_{Ae}}{C_{Ao} - C_{Ae}} \quad (23)$$

which also means that the two models are not similar.

Consequently:

- the fluid-phase mass transfer Biot number Bi_{Mf} given by Eq. (22) is not analogous to the heat transfer Biot number Bi_H despite the formal similarity between them and
- Bi_{Mf} does not have the same role for characterizing transient mass transfer as Bi_H has for transient heat transfer.

Thus the equality of Bi_H and Bi_{Mf} does not mean the similarity of the two transient processes. This shows that the simple change of heat transfer parameters to mass transfer parameters of the heat transfer Biot number does not give an analogous mass transfer Biot number. That practice is applicable for all the dimensionless terms used except the Biot number.

For drying grains it was found that the moisture content difference within the particle can be neglected if Bi_{Mf} is under 2 when the bulk air temperature and air humidity was 80 °C and 10 g/kg, respectively. (PARTI and DUGMANICS, 1989). For different air characteristics various limits of the fluid-phase mass transfer Biot number were obtained which support the statement that Bi_{Mf} is not analogous to Bi_H .

Solid-Phase Mass Transfer Biot Number

NEWMAN (1931) assumed that in the second stage of drying the evaporation rate is proportional to the free moisture content at the surface (at first suggested by LEWIS, 1921). That assumption has been extensively used in the theory of drying and has been found especially useful to describe the drying process of different agricultural grains (see e.g. PATIL, 1988; WALTON et al., 1988). Extending Newman's concept on general mass transfer, Eq. (7) should be substituted by Eq. (24):

$$-D \frac{\partial C_A}{\partial x} \Big|_{x=L} = k_C (C_{As} - C_{Ae}) . \quad (24)$$

Eq. (24) is analogous to Newton's law of cooling in which, instead of the fluid-phase driving force ($c_{As} - c_{Ab}$) and the fluid-phase mass transfer coefficient k_c , a hypothetical solid-phase driving force ($C_{As} - C_{Ae}$) and a solid-phase mass transfer coefficient k_C are used.

Using the dimensionless variables *Eq.* (24) becomes:

$$-\frac{\partial \nu}{\partial \eta} \Big|_{\eta=1} = \frac{k_C L}{D} \nu(1, F_{oM}) . \quad (25)$$

Comparing *Eq.* (25) to the relevant equation on heat transfer, *Eq.* (14), it can be seen that they are analogous and a mass transfer Biot number can be defined as follows:

$$Bi_{Ms} = \frac{k_C L}{D} \quad (26)$$

which can be considered the solid-phase mass transfer Biot number. This is like the dimensionless term introduced by Newman and used by Luikov as the mass transfer Biot number for drying (LUIKOV, 1966). That solid-phase mass transfer Biot number is formally not similar to the heat transfer Biot number since the solid-phase mass transfer coefficient instead of the convective mass transfer coefficient substitutes the convective heat transfer coefficient but it gives similarity between the two processes.

Consequently:

- the solid-phase mass transfer Biot number Bi_{Ms} described by *Eq.* (26) is analogous to the heat transfer Biot number Bi_H despite the formal dissimilarity between them and
- Bi_{Ms} has the same role in transient mass transfer as Bi_H has in transient heat transfer.

Thus the equality of Bi_H and Bi_{Ms} does mean the similarity of the two transient processes. It follows that if Bi_{Ms} is less than 0.1 the mass transfer resistance inside the body can be neglected and a lumped parameter model can be used for describing the mass transfer process. A distributed parameter model should be used if Bi_{Ms} is larger than 0.1.

However, the application of the solid-phase mass transfer Biot number is limited since the assumption used in *Eq.* (24) is not valid generally for the whole range of the mass transfer process. This means that the application of the solid-phase mass transfer Biot number is confined to that range of the concentration change where *Eq.* (24) is valid. As was mentioned *Eq.* (24) has proved useful and applicable e.g. for drying agricultural grains since they usually dry in the falling rate period. For drying of other materials the solid-phase mass transfer coefficient k_C was found to depend on the moisture content of the material, the temperature and the humidity of the air, etc. (LIKOV, 1952). The drying process of these materials can be described very imprecisely by *Eq.* (24), consequently Bi_{Ms} cannot be used for these drying processes.

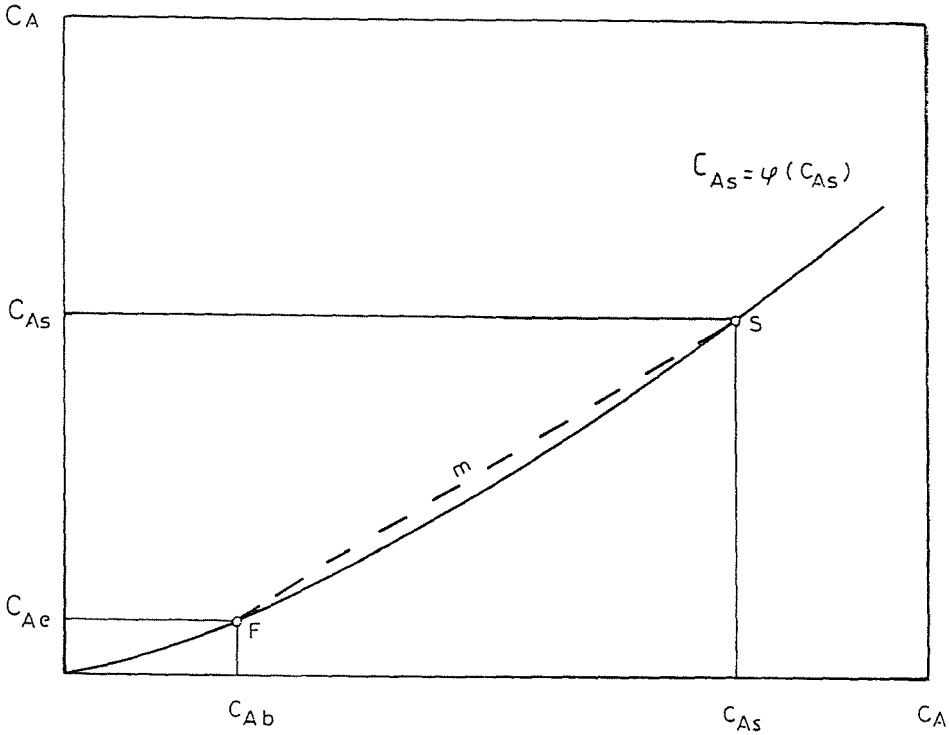


Fig. 2. The isothermal equilibrium curve and its replacement

Analogous Mass Transfer Biot Number

The equilibrium conception will be used in the following to get similarity between heat transfer and mass transfer and to get the generally usable mass transfer Biot number. The equilibrium curve described by Eq. (2) is shown in Fig. 2. The condition at the surface is represented by point S and point F represents the final concentrations. From the geometry of the figure:

$$c_{As} - c_{Ab} = m(c_{As} - C_{Ae}), \tag{27}$$

where m is the slope of the chord SF . Substituting Eq. (27) into Eq. (15) we get:

$$-\frac{\partial v}{\partial \eta} \Big|_{\eta=1} = \frac{k_c L}{D} \frac{c_{As} - c_{Ab}}{C_{As} - C_{Ae}} \frac{C_{As} - C_{Ae}}{C_{Ao} - C_{Ae}} = \frac{k_c L}{D} m \nu (1, Fo_M). \tag{28}$$

Eq. (28) is analogous to Eq. (14) if the mass transfer Biot number is defined as:

$$Bi_M = \frac{k_c L}{D} m = m Bi_{Mf} . \quad (29)$$

This can be described into the next form, using Eqs. (7) and (27) as well as Fig. 1

$$Bi_M = m Bi_{Mf} = \frac{C_{Ac} - C_{As}}{C_{As} - C_{Ae}} \quad (30)$$

which is similar to Eq. (17).

The mass transfer Biot number introduced in Eq. (29) is analogous to the heat transfer Biot number because it gives similarity between the two transport processes. Their formal similarity is altered by the slope of the equilibrium curve in the relevant concentration range. Therefore this will be called the analogous mass transfer Biot number (or simply mass transfer Biot number). It follows that the mass transfer Biot number defined by Eq. (29) is applicable for utilizing the statements and results of the transient heat transfer for transient mass transfer by replacing Bi_M for Bi_H . It means that e.g. if $Bi_M < 0.1$ the diffusion can be described by a lumped parameter model and a distributed parameter model should be used if Bi_M is above that limit. The investigation showed that temperature equilibrium can be assumed if $Bi_H > 100$. Consequently, surface moisture content equilibrium can be assumed if $Bi_M > 100$ (PARTI, 1992). These assumptions are quite frequently used for drying agricultural grains. To assume temperature equilibrium means that the effect of the temperature change of the material to be dried is ignored in the drying process. Accordingly the drying process is considered isothermal at the bulk air temperature.

It can be shown that the modified mass transfer Biot number introduced earlier by a trial-and-error method for investigating the applicability of the different models and assumptions on drying can perform the same function as the mass transfer Biot number defined by Eq. (29) (PARTI, 1992).

Finally if Eq. (24) is valid a relation can be obtained between the three mass transfer Biot numbers by comparing Eq. (22) and Eq. (29) as we get:

$$Bi_M = m Bi_{Mf} = Bi_{Ms} . \quad (31)$$

Application

It was mentioned that for modelling drying agricultural grains temperature equilibrium is assumed frequently. It follows that the kernel temperature reaches the bulk air temperature immediately at the beginning of

the process. Then for describing the process only the investigation of the moisture content change in time (that is: the mass transfer) is required. The drying curve (or the average concentration curve) of such isothermal drying is shown in *Fig. 3* for two bulk air temperatures, $t_b = 80\text{ }^\circ\text{C}$ and $t_b = 40\text{ }^\circ\text{C}$. The value of the mass transfer Biot number which gives similarity is $Bi_M = 193$ when the values of the fluid-phase mass transfer Biot number are $Bi_{Mf} = 200$ and $Bi_{Mf} = 623$, respectively. The two drying curves coincide as can be seen. Therefore the equality of Bi_M gives similarity between the two processes (here between the two drying processes), although the fluid-phase mass transfer Biot numbers differ significantly. The heating curve (the average dimensionless temperature curve) is displayed in *Fig. 4* for $Bi_H = 10$ along with several experimental data for adsorption when $Bi_M = 10$. The slope of the isothermal equilibrium curve was changing slightly so $Bi_M = 10$ is an average value. Of course, *Fig. 3* and *Fig. 4* do not verify the similarity proved mathematically, they illustrate it only.

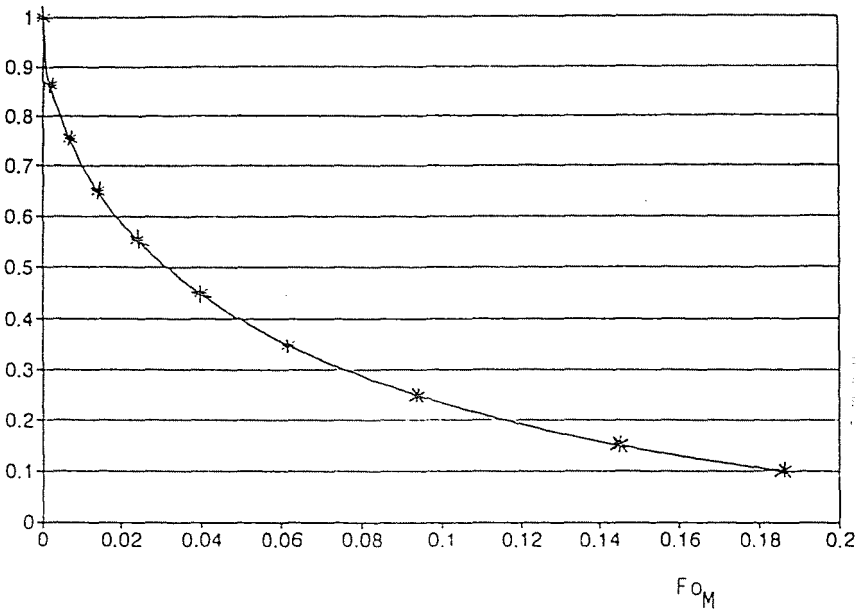


Fig. 3. Drying curves for two bulk air temperatures — $t_b = 80\text{ }^\circ\text{C}$; ** $t_b = 40\text{ }^\circ\text{C}$

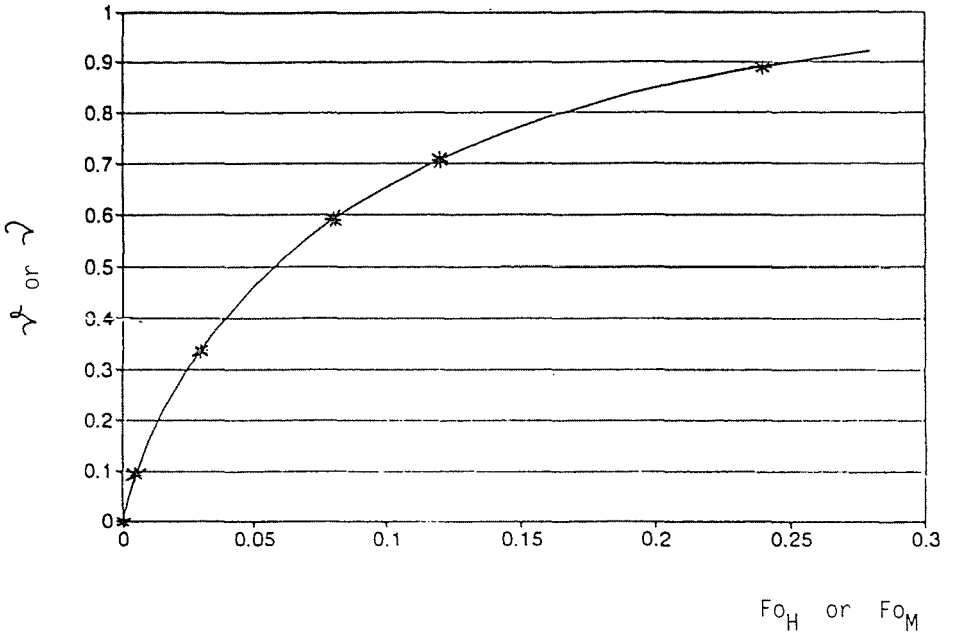


Fig. 4. Heating curve and adsorption experimental data — heating ; ** adsorption

Conclusions

A simple model is used in the present paper to derive the mass transfer Biot numbers, namely the fluid-phase, the solid-phase and the so called analogous mass transfer Biot number. For heat transfer only one Biot number can be derived. This difference between the two transport processes is generated by the difference at the interface: there is equilibrium at the interface in both cases but for heat transfer it means equality of the temperatures of the two phases when for mass transfer it means only a unique functional relationship of the two concentrations (if there is no contact resistance at the interface).

It was shown that:

- the fluid-phase mass transfer Biot number is formally similar to the heat transfer Biot number but despite the formal similarity it does not give similarity between mass transfer and heat transfer;
- the solid-phase mass transfer Biot number is formally dissimilar to the heat transfer Biot number but gives similarity between mass transfer and heat transfer. Its use is limited significantly, since Newman's

- concept on the boundary condition is generally not applicable for the whole mass transfer process;
- the fluid-phase mass transfer Biot number modified by the slope of the equilibrium curve in the appropriate concentration range or the analogous mass transfer Biot number gives similarity between mass transfer and heat transfer and has a general applicability. All the results and statements on heat transfer are applicable for mass transfer substituting the heat transfer Biot number by this mass transfer Biot number.

Nomenclature

Bi	Biot number
c	fluid concentration
C	solid concentration
D	diffusion coefficient
Fo	Fourier number (dimensionless time)
h	convective heat transfer coefficient
k	thermal conductivity
k_c	convective (fluid-phase) mass transfer coefficient
k_C	solid-phase mass transfer coefficient
L	geometrical size (half thickness of the slab)
m	slope of the equilibrium curve (see <i>Fig. 2</i>)
t	fluid temperature
T	solid temperature
x	coordinate

Greek letters

α	thermal diffusivity
η	dimensionless coordinate
ϑ	dimensionless temperature
ν	dimensionless concentration
φ	function
τ	time

Subscripts

A	diffusing component
b	bulk value
c	at the symmetry plane
e	equilibrium value
f	fluid phase

f	final value
H	heat transfer
M	mass transfer
o	initial value
s	solid phase

References

- BRUIN, S. - LUYBEN, K. CH. A. M. (1980): Drying of Food Materials: A Review of Recent Developments. pp. 155-215. In: Mujumdar, A. S. (ed.): *Advances in Drying*. Vol 1. Washington, Hemisphere Publishing Corporation.
- IMRE, L. - NÉMETH, J. (1974): Designing Adsorption Equipment. pp. 441-482. In: Imre, L. (ed.): *Handbook of Drying* (in Hungarian). Budapest, Műszaki Könyvkiadó.
- KEEY, R. B. (1972): *Drying: Principles and Practice*. Oxford, Pergamon Press.
- LANGHAAR, H. L. (1951): *Dimensional Analysis and Theory of Models*. John Wiley, New York.
- LEWIS, W. K. (1921): The Rate of Drying of Solid Materials. *The Journal of Industrial and Engineering Chemistry*, Vol. 13, No 5, pp. 427-432.
- LIKOV, A. V. (1952): *Theory of Drying* (in Hungarian). Nehézipari Könyvkiadó. Budapest.
- LUIKOV, A. V. (1966): *Heat and Mass Transfer in Capillary-Porous Bodies*. Oxford, Pergamon Press.
- MEIERING, A. G. - HOFFMANN, O. B. - BAKKER-ARKEMA, F. W. (1970): Ein Beitrag zur numerischen Behandlung des koppelten Stoff- und Wärmeaustausch bei der Trocknung von Frischkompost. *Grundlagen der Landtechnik*, Vol. 20, No 3, pp. 71-76.
- NEWMAN, A. B. (1931): The Drying of Porous Solids: Diffusion and Surface Emission. *Transaction of the American Institute of Chemical Engineering*, Vol. 27, pp. 203-220.
- PAKOWSKI, Z. - MUJUMDAR, A. S. (1987): Basic Process Calculations in Drying. pp. 83-129. In: Mujumdar, A. S. (ed.): *Handbook of Industrial Drying*. New York, Marcel Dekker.
- PARTI, M. - DUGMANICS, I. (1989): Significance of Intraparticle Heat and Mass Conduction on Drying. *Hungarian Journal of Industrial Chemistry*, Vol. 12, No. 2, pp. 235-243.
- PARTI, M. (1990): Selection of Thin-Layer Drying Model for Agricultural Grain. *International Conference on Agricultural Engineering AgEng'90*, Berlin, pp. 310-311.
- PARTI, M. (1992): Similarity of Heat and Moisture Transfer. pp. 125-145. In: *Drying of Solids*. Ed. Mujumdar, A. S. Oxford and IBH Publishing Co., New Delhi.
- PATIL, N. D. (1988): Evaluation of Diffusion Equation for Simulating Moisture Movement within an Individual Grain. *Drying Technology*, Vol. 6, No. 1, pp. 21-42.
- SCHLÜNDER, E. U. (ed). (1983): *Heat Exchanger Design Handbook*. Vol. 2. Fluid Mechanics and Heat Transfer. Washington, Hemisphere Publishing Corporation.
- TREYBAL, R. E. (1968): *Mass Transfer Operations*. New York, McGraw-Hill Book Company.
- WALTON, L. R. - WHITE, G. M. - ROSS, I. J. (1988): A Cellular Diffusion-based Drying Model for Corn. *Transaction of ASAE*, Vol. 31, No. 1, pp. 279-283.