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Elastic-plastic Analysis of Bonded Composite Repair in Cracked Welded Structures

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Abstract

The present paper employs the 3D finite element method to investigate the elastic-plastic behavior of repaired cracks in welded structures reinforced with bonded composite patches. The analysis is based on the *J*-integral, an important parameter for crack tip characterization. Two different patch configurations are considered: single and double symmetric patches. The effect of patch thickness, adhesive properties and the mechanical behavior of weld-metal on the *J*-integral variation are systematically evaluated. The results revealed that the double symmetric patch configuration excels over a single patch in terms of improving repair efficiency. Additionally, it is found that the mechanical behavior of weld-metal plays a crucial role in determining the performance of the reinforced welded structures.

Keywords

bonded composite repairs, patch, weld cracks, elastic-plastic behavior, *J*-integral, base metal, weld metal

1 Introduction

Crack repair in welded structures is a highly relevant and important field of investigation given its extensive application in construction and manufacturing industries. Effective repair techniques are required to maintain the integrity and safety of welded structures since welds are known to be potential sources for crack initiation and propagation. One common repair technique is the use of bonded composite patches, which are made of advanced composite materials and are applied to the surface of the cracked structures. These patches can provide reinforcement and help to distribute the stress around the crack, thus reducing the crack growth. Several studies have investigated the influence of the patch properties on the efficiency of repaired structures. They have also investigated the use of different patch configurations, such as single and double symmetric patches. The use of bonded composite materials for crack repair is gaining increased attention due to their high strength-to-weight ratio [1], excellent durability, and ease of handling. Therefore, the research in this field can lead to development of more efficient and cost-effective crack repair techniques, which can have a significant impact on the safety and reliability of welded structures.

The use of bonded composite patches involves applying a patch to the surface of the metallic panel or structure in the area of the crack. The patch is then bonded to the metal using a specially designed adhesive. Important efforts made by many researchers [2–8] led to a large development of repairing techniques by bonded composite patches.

According to Walde [9] and Lilhare and Srilakshmi et al. [10], the presence of the patch doubles the fatigue life. Khalid [11] investigated how adhesive performance affected the effectiveness of repairs. A 65% reduction in Stress Intensity Factor (SIF) was seen in the presence of the patch during fatigue tests on single-edge samples that had been patched and those that had not. Alnaser et al. [12] investigated the crack propagation behavior on a pressure vessel, using two types of repair, a composite patch and full-encirclement. It was found that the two repair types have similar crack propagation behavior during fatigue testing. Albedah et al. [13] analyzed, experimentally and numerically, the fatigue crack behavior in aluminum plates repaired with a composite patch bonded with adhesives, based on fatigue life calculations and J-integral method. They found that the patch improves fatigue life, but that this improvement decreases considerably with increasing applied fatigue load. The Al 2024 T3 presents better resistance to fatigue crack propagation than the AA 7075 T6, in both repaired and unrepaired cases.

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The finite element method has greatly enhanced the study of fracture behavior in the presence of patch repairs in recent times. Benachour et al. [14] examined the effectiveness of boron-epoxy patches on crack behavior in aluminum plates and found that using both one-sided and two-sided patches significantly extended the life cycle. Several studies have focused on repairing cracks in thin plates [15–21], utilizing FE methodology to investigate crack patching behavior. Colombi et al. [22] conducted FEM analysis on the use of pre-stress composite patches to reinforce broken steel components, resulting in a significant extension of the life cycle. Seo et al. [23] made a comparison of fatigue crack growth behavior considering different plate thicknesses in the presence of composite patch repair. The difference in SIF increased by increasing the patch thickness.

For three-dimensional problems, Boulenouar et al. [24] conducted a study on the repair of semicircular cracks in finite-thickness plates. Merzoug et al. [25] used numerical analysis to investigate the behavior of repaired surface cracks with bonded composite patches. Maligno et al. [26] utilized a three-dimensional finite element approach to study the behavior of fatigue crack growth in a thick aluminum alloy plate that was repaired with a bonded composite patch. In the study conducted by Mhamdia et al. [27], the effects of patch geometry on the efficiency of bonded composite repair were examined, and the results indicated that the H shape exhibited the best performance. Benyahia et al. [28] investigated the impact of patch geometry on the performance of three-dimensional crack repair under mechanical and thermal loading. Ramji et al. [21, 29] suggested that unsymmetrical patch shapes did not have a significant impact on SIF reduction for mixed-mode problems. Ivanez and Braun [30] analyzed the performances of circular single- and double-sided composite patch repairs by computing the maximum SIF of a repaired surface crack.

In the case of welded joints, the weld metal has a mismatch in tensile properties from the surrounding base material. The term 'mismatch' then refers to the two materials having different yield stress and hardening exponents in a Ramberg-Osgood law or having different stress-strain relationships. The study of the repair elastic performance can be done by the evaluation of SIF using different existing methods [31–38]. However, this paper's focus is on analyzing the elastic-plastic behavior of a repaired crack in a mismatched weld by computing the *J*-integral parameter around the crack-tip, using the finite element method. The weld model considered is an idealized bi-material 'sandwich' structure without Heat Affected Zone (HAZ) and residual stress. The two materials, weld and base metals have the same elastic modulus E and Poisson's ratio. The investigated bonded composite patch is the Graphite/Epoxy, which have a very successful use according to Baker's team [2]. The patch and the adhesive characteristics are the factors influencing how effectively bonded composite repairs perform in practice, according to several authors [39–43]. Accordingly, the present study investigates how changes in the Young's modulus of the adhesive, adhesive thickness, patch thickness, and the elastic-plastic properties of the weld-metal impact the evolution of *J*-integral values for the repaired cracks.

2 Geometrical model

The geometry of the cracked plate considered in this study is shown in Fig. 1. The plate has a length of $2L = 448$ mm, width of $2W = 192$ mm, thickness, $th = 4$ mm and a center-crack of length $2a$. The crack is located inside the weld material which has width of $2h = 40$ mm. The length of the uncracked ligament is defined as $2l$, where, $l = W-a$. To study its elastic-plastic behavior, the following simplifying assumptions were made:

1. The elastic properties of base and weld materials are the same (i.e., same elastic modulus E, and Poisson's ratio). This assumption is supported by experimental data [44–51]. The uniaxial stress-strain curves for the base or weld materials follow the Ramberg-Osgood equation:

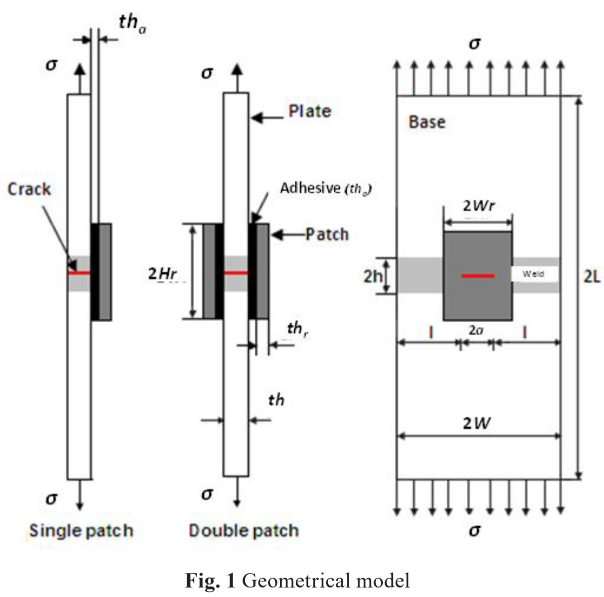

Fig. 1 Geometrical model

$$\frac{\varepsilon}{\varepsilon_{0k}} = \frac{\sigma}{\sigma_{0k}} + \alpha_k \left(\frac{\sigma}{\sigma_{0k}} \right)^{n_k} , \quad (1)$$

where, σ_{0k} is a reference stress, which can be arbitrary, but usually assumed to be the yield stress σ_{yk}, E is the modulus of elasticity, $\varepsilon_{0k} = \sigma_{0k}/E$ is the associated reference strain, and n_k and α_k are model parameters usually chosen from a best fit of actual data, and $k = 1$ or 2 representing base or weld materials, respectively.

2. Only base and weld metals are considered to represent the anisotropy of the material, at the level of plastic behavior. In this analysis, the HAZ is not considered.
3. Weld-induced residual stresses are ignored. This is a reasonable assumption when the stresses due to primary loads in a structure are much larger than the residual stresses. Otherwise, residual stresses should be considered, but in that case the *J*-integral proposed by Rice [52] may not be a meaningful fracture parameter due to the loss of path independence.

The central crack is repaired with single and double Graphite/Epoxy patch of dimensions: $Hr = Wr = 140$ mm, and $th_r = 2$ mm, the plies in the patch had a unidirectional lay-up, meaning that the fibers are orientated parallel to the stress and the specimen length directions. The adhesive used to bond the patch on cracked plate is FM73, epoxy adhesive. The adhesive thickness (th_a) is taken equal to 0.2 mm. $\sigma = 130$ MPa is the applied load (Fig. 1).

The material properties used in this study are given in Table 1, with base metal and two weld materials, taken from a previous work of Lei and Ainsworth [53].

3 Finite element modeling

A 3D-FEM method using ANSYS APDL is carried out for the analysis. The FE model consists of four parts to model the base metal, the weld metal, the adhesive and the composite patch, the considered volumes are then combined and overlapped into one volume. Only one-fourth of the repaired plate was taken into consideration for reasons of symmetry. The plate, consisting of four layers of elements in the thickness direction, was taken into account, while the adhesive and composite patch had one and two layers of elements through the thickness, respectively. The contact between the plate/adhesive and between the adhesive/composite is assumed perfect, with common nodes between layers. For this purpose, the mesh size must be with same dimensions for all the common parts of materials.

The mesh refinement around the crack-tip was achieved using SOLID185 brick elements. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions. These elements with dimensions of 0.03 mm are considered small enough to accurately describe the deformation and stress gradients. To reduce the computational costs, coarse mesh is applied in the remaining regions. It is worth mentioning that the mesh size is uniform in the common part of the materials. Fig. 2 shows the overall mesh of the cracked plate and mesh refinement in the crack-tip region.

4 Analysis and results

In the present work, the 3d-FEM method is used to compute the *J*-integral contour of repaired cracks in weld, with bonded composite for two configurations of the patch: single and double symmetric. In this study, the *J*-integral

Table 1 Mechanical properties

Property	Base Metal BM	Weld Metal (WM) WM 1	WM 2	Composite patch Graphite/Epoxy	Adhesive FM73
E_1 (MPa)	207000	207000	207000	134000	2550
E_2 (MPa)	-	-	-	10300	-
E_3 (MPa)	-	-	-	10300	-
G_{12} (MPa)	-	-	-	5500	-
G_{13} (MPa)	-	-	-	5500	-
G_{23} (MPa)	-	-	-	3200	-
v_{12}	0.3	0.3	0.3	0.33	0.32
v_{13}	-	-	-	0.33	-
v_{23}	-	-	-	0.53	-
Yield stress σ_0 (MPa)	413.68	1241.04	496.42	-	-
α	1.12	0.37	0.93	-	-
n	9.71	9.71	9.71	-	-

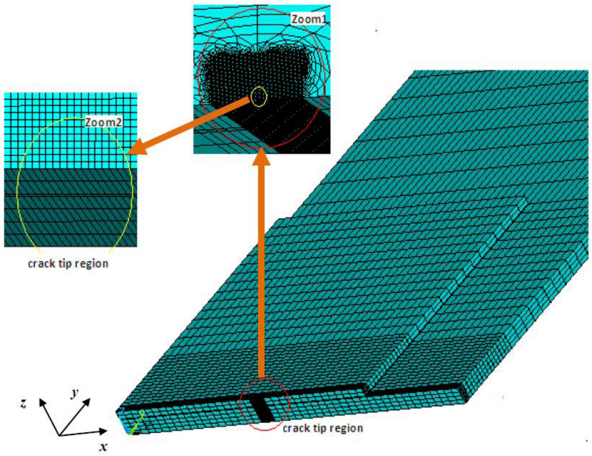

Fig. 2 Typical mesh model of the quarter of the structure and near the crack-tip

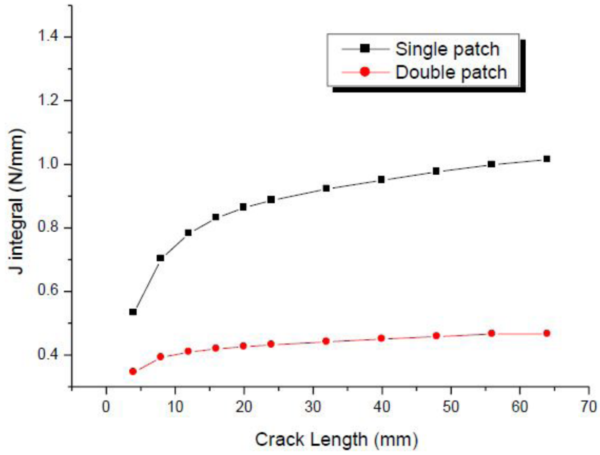

Fig. 4 Comparison of the *J*-integral between single and double patched cracks

proposed by Rice [52] is taken from the mid-side point of the plate thickness located at the crack front.

4.1 Comparison between repaired and unrepaired cracks

Fig. 3 presents the evolution of the *J*-integral values as a function of the crack length under mode I loading, for repaired and unrepaired cracks using a single patch. The beneficial effect of the patch is clearly evident as the *J*-integral value decreases significantly. This reduction is due to the fact that the patch absorbs the efforts transmitted by the plate through the adhesive especially for high crack lengths. The maximum reduction of the *J*-integral reaches 96.6 %.

4.2 *J*-Integral for double and single composite repair configurations

Fig. 4 presents the evolution of the *J*-integral as a function of the crack length for single and double patch techniques.

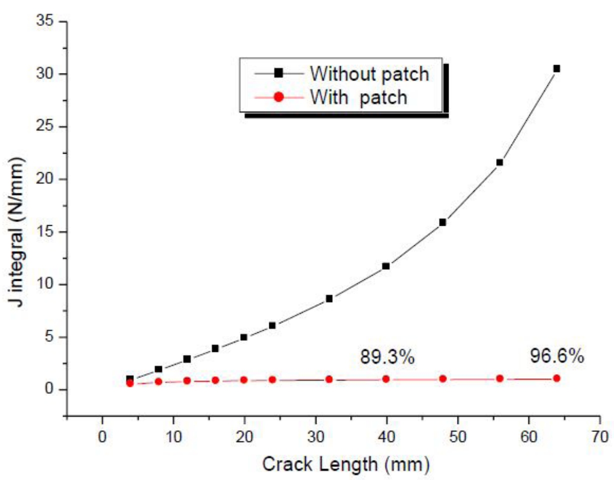

Fig. 3 Comparison of the *J*-integral between repaired and unrepaired cracks

One can see that when the crack length equals 12 mm, the *J*-integral appears to be stabilizing for the case of the double patch repair. This behavior was observed in the elastic [54, 55] and elastic-plastic analysis [56, 57]. In contrast, by increasing the crack length, the value of the *J*-integral keeps increasing for the case of single patch repair.

We can also report that the reduction of *J*-integral values is less important for the single patch repair compared to the double patch repair where the reduction rate can be estimated at 54%. The decrease is attributed to the fact that the absorption of stresses by the double patch configuration is increased to twice. This effect was also reported by Bouchelarm et al. [40].

4.3 Effect of the adhesive properties

It's well recognized that the adhesive characteristics have a significant role in the repair procedure involving a bonded composite patch. The adhesive layer facilitates the transmission of stresses from the cracked plate to the composite patch, with the most stress being transmitted from the crack area. In this section, the impact of both adhesive thickness and its Young's modulus on the *J*-integral is examined for both single and double patch configurations.

4.3.1 Effect of the adhesive thickness (th_a)

Fig. 5 presents the evolution of *J*-integral as a function of the crack length for various values of the adhesive thickness (th_a). It is evident that a decrease in adhesive thickness causes a decrease in the *J*-integral, indicating that a lower adhesive thickness is preferred for crack repair. This effect was highlighted by [54, 55] in the case of elastic analysis, and by [56, 57] in the case of elastic-plastic analysis. High thickness improves adhesion but reduces the transfer of loads to the patch, decreasing the patch's positive

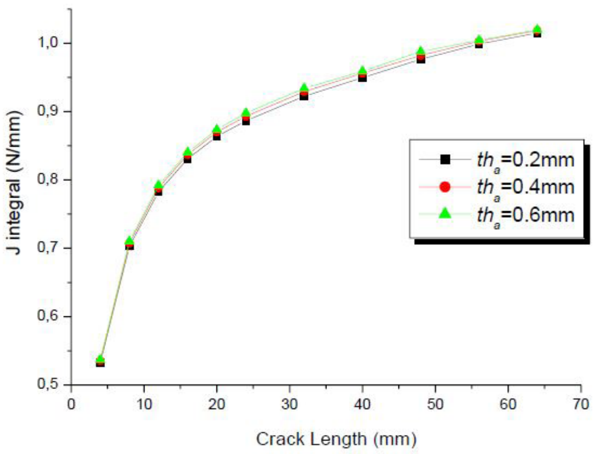

Fig. 5 Effect of the adhesive thickness on the variation of the *J*-integral

advantages. Conversely, a thinner adhesive layer improves the transmission of load to the patch but increases the possibility of the adhesive failure. In our case; it is useless to increase the adhesive thickness beyond 0.4 mm since the reduction obtained does not exceed 1% when the thickness (th_a) increases from 0.2 mm to 0.4 mm. This small reduction is due to the fact that the strong stress are already absorbed by the first repair, with th_a = 0.2 mm.

Fig. 6 illustrates the variation of the *J*-integral as a function of the adhesive thickness for cracks repaired with single and double symmetrical patches. The results indicate that the *J*-integral remains almost independent of adhesive thickness when the thickness (th_a) varies between 0.1 and 0.4 mm. Furthermore, the use of a double patch leads to a significant reduction in the *J*-integral, reaching up to 54% compared to the single-patch repair.

4.3.2 Effect of the adhesive Young's modulus (*Ea*)

It is well recognized that premium adhesives are distinguished by their lower Young's modulus values, aiming to minimize the stress intensity within the adhesive layer. When dealing with bonded composite repairs, the adhesive's main function is to efficiently transfer stresses. In theory, employing an adhesive with a higher Young's modulus would be advantageous for the repair procedure.

The effect of the Young's modulus of the adhesive is illustrated in Fig. 7. This one shows the variation of the *J*-integral according to the adhesive Young's modulus for a crack repaired with single patch. The effect of this module is marked more where we note a reduction in the *J*-integral of about 2.5% when the Young's modulus increases from 2550 to 4000 MPa. One can conclude that in theory, a rigid adhesive performs better in a crack repair even if the reduction rates seem to be small. However, to guarantee an improved stress transmission to the patch and prevent adhesive failure caused by higher stresses in the adhesive layer, the adhesive must be properly chosen. In fact, higher stiffness adhesives significantly increase the performance of the patch repair, but for the adhesive itself, the stresses increase and this is not desirable [58].

Fig. 8 illustrates the variation of the *J*-integral for a crack repaired using the two techniques as a function of adhesive Young's modulus (*Ea*). It can be observed that the *J*-integral is independent of the variation in adhesive Young's modulus. The results also highlight the advantageous effect of the double-patch configuration, with the reduction in the *J*-integral estimated to be approximately 53% relative to the single-patch repair.

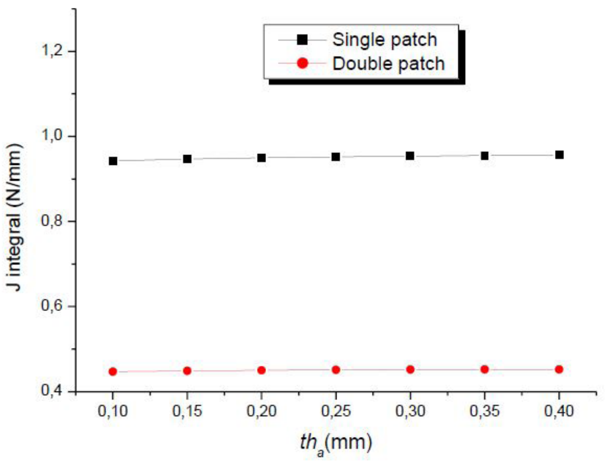

Fig. 6 Comparison of the *J*-integral between single and double patched cracks, for a = 40 mm

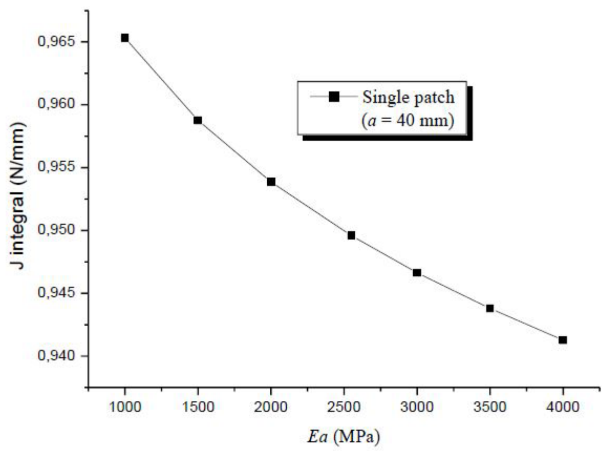

Fig. 7 Effect of the adhesive Young modulus on the variation of the *J*-integral

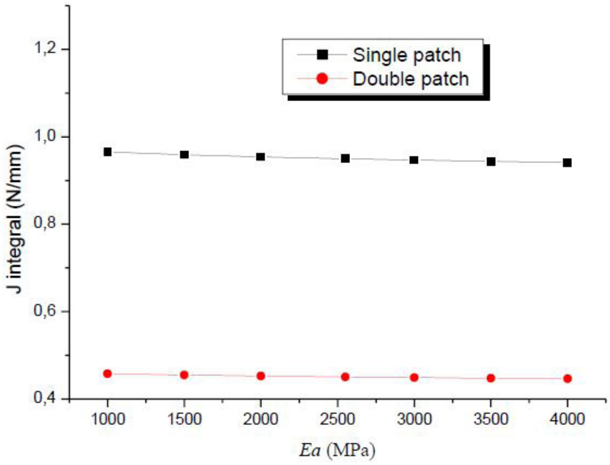

Fig. 8 J-integral for single and double composite repair configurations, for a = 40 mm

4.4 Effect of the patch thickness (th_r)

The mechanical properties of the patch and the adhesive, as well as their geometric properties like patch thickness, affect the durability of previously damaged structures and the degree of stresses around the crack tip. The plot of the J-integral variation in Fig. 9 illustrates this influence of the patch thickness, for cracks repaired with single and double symmetric patches, as a function of the patch thickness (th_r). Clearly, we note that an increase in this geometric parameter leads to a reduction of J-integral values for the two repair modes, and this is desirable. In other words, thicker patches are better for reducing the stresses highly concentrated at the crack tip. It is therefore preferable to use thicker reinforcements to improve the performance of repaired structures. However, the reinforcement material should not be too much thick in order to preserve all the beneficial effects of the repair without penalizing the overall structure weight. Furthermore, in some situations, especially for aerodynamic considerations, a thicker patch is not desirable. Finally, and according to the results, one can notice that for a better distribution of the stresses, it is possible to use a double patch repair with low thickness and still keep a better stress reduction than a thicker single patch repair.

4.5 Effect of the elastic-plastic behavior of weld-metal

In order to analyze this effect, the variation of the J-integral as a function of the crack length is presented in Fig. 10 for two types of weld metals (WM1 and WM2) using a single patch repair.

The beneficial effect of repair is definitely visible for the two welded structures, but with different reduction rates. It is related to the fact that the local stress field for WM1 is larger than that obtained for WM2. In addition, the maximum reduction of the J-integral in weld metal WM1 can be estimated at 23.8% compared to that obtained for weld metal WM2. Given that the weld metals WN1 and WN2 exhibit identical elastic behavior, it can be concluded that the difference of plastic behavior between WN1 and WN2, characterized by the parameters σ_0 and α (with $n = cst$ in the present case), plays a key role in the effectiveness of patch repair in welded structures.

5 Conclusion

In this study the three-dimensional finite element method has been used to analyze the advantage of using bonded composite patches to repair cracks in welded structures by taking in consideration the influence of various parameters.

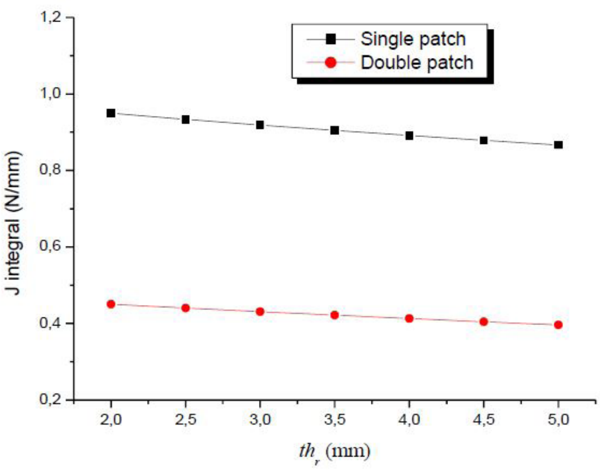

Fig. 9 Effect of the patch thickness on the evolution of the J-integral, for a = 40 mm

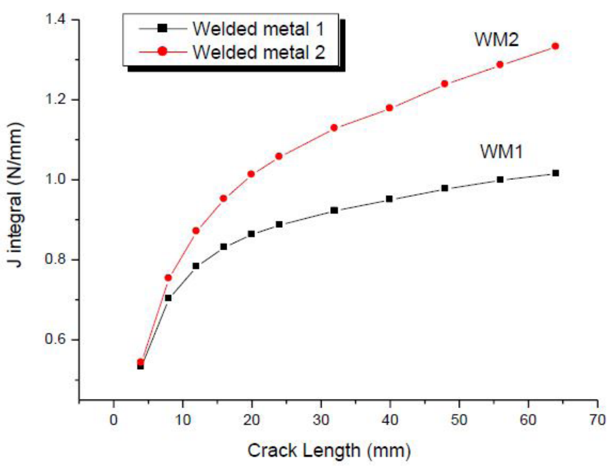

Fig. 10 Effect of the behavior law of welded metal on the variation of the J-integral

The effects of adhesive Young's modulus, adhesive thickness, patch thickness and elastic-plastic behavior of weld-metal on the variations of the *J*-integral values of the repaired cracks were investigated. The obtained results enable us to draw the following conclusions.

- The presence of the patch contributes to the stress reduction at the crack tip due to a transfer of stresses through the adhesive towards the patch. The bonded patch considerably reduces the *J*-integral values at the crack-tip by 95%.
- When compared to the single patch, the double symmetric patch improves repair performance considerably. The reduction of *J*-integral values can reach 54%.
- While it may be theoretically advantageous to opt for adhesives with a high Young's modulus in order to significantly reduce *J*-integral values, the selection of adhesives for crack repairs must be made carefully. Increased Young's modulus significantly improves patch performance. However, it also puts more stress on the adhesive layer, increasing the possibility of patch failure.
- The thickness of the bonded composite patch and the *J*-integral are inversely proportional. To preserve the benefits of the repair and prevent patch failure, the thickness must always be carefully chosen.
- Finally, the type of the weld metals is an important parameter which affects the patch repair behavior in the welded structures.

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