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# Axisymmetric Stress State of Adhesive Joint of a Circular Patch with a Plate Weakened by a Circular Cut-out

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**Abstract**

Adhesive lap joints are widely used in modern structures. Known analytical mathematical models of the stress state of lap joints describe the joints of straight beams or cylindrical coaxial pipes. It is assumed that the stress state of these structures depends on only one coordinate. The study of the stress state of plates with defects, which are reinforced by patches, in most cases requires the use of at least two-dimensional mathematical models. In this work, it is shown that the axial symmetry of the plate, the cut-out, the patch, and the applied load makes it possible to reduce the problem to a one-dimensional problem in the polar coordinate system. An adhesive lap joint with circular symmetry is considered for the first time. The stress-strain state of the structure is described in an analytical form. Comparison of the results of calculating the stress state of the joint with the results of finite element modelling showed good adequacy of the proposed mathematical model.

**Keywords**

adhesive joints, analytical models, interlaminar stresses, plates

## 1 Introduction

In operation, thin-walled structures can acquire different defects and damages. As a rule, the repair procedures of such damages consist in making the hole rounded (to reduce stress concentration) and reinforcing the cut-out, e.g., using a repair patch. The patch can be joined with the main plate over the entire patch surface (an overlap adhesive joint), along lines (welded joints) or by a system of points (rivet and bolt joints). Adhesive joints come with tightness, high aerodynamic performance, and technological effectiveness and do not disturb the integrity of the structure of fibrous composites. As a rule, finite-element modelling is used to investigate the stress state of plates having defects reinforced with repair patches. The patch can have an octagonal [1], circular [2], and rectangular [3] form. In addition to numerical simulation, experimental studies of joints [4] and reinforced plates for strength [5] are also carried out.

Known analytical solutions to the problem of the stress state of lap joints consider either the joint of beams [6, 7], or the joint of coaxial pipes [8]. A detailed review of existing analytical models is given in papers [9] and [10]. Two-dimensional mathematical models of joints that account for non-uniform distribution of stresses over the joint width also assume a rectangular adhesive bond form. In papers [11] and [12], the problem of the joint stress state is reduced to a system of differential equations in partial derivatives with respect to displacements, which is solved numerically by the finite difference method. If we simplify the mathematical model, assuming that the connected plates are shear compliant, then the analytical solution can be obtained in the form of double Fourier series [13] or as an expansion of the series in terms of eigenfunctions [14]. There is a second approach that allows one to describe the stress state of a joint in an analytical form. It lies in the fact that the transverse deformations, which are due to Poisson's ratios, are assumed to be equal to zero. This approach makes it possible to find in an analytical form the stress state of plate joints under non-uniform loads [14], joints of plates with different widths, and the solution of some other problems [15]. There are more simple two-dimensional models of the stress state of joints of rectangular plates [16].

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In most of the above works, the distribution of stresses over the thickness of the plates or shells being joined is assumed to be uniform or linear one. There are two-dimensional in thickness mathematical models of the stress state of overlap adhesive joints based on the solution to a two-dimensional problem of the theory of elasticity. In the paper [17], the problem of the axisymmetric stress state of the joint of the plate, which contains a through thin defect, with a circular patch is solved. In this case, transverse displacements due to the bending of the structure are not taken into account.

If we assume that the distribution of stresses over the thickness of the joined plates is uniform one (without account for bending), and the joint of the patch with the main plate is not distributed over the surface but concentrated along the line (a welded joint), then this problem statement has an analytical solution for a non-axisymmetric load [18, 19].

The objective of this work is to build an analytical solution of the problem on the axisymmetric stress state of a plate that has a circular cut-out reinforced on one side with a concentric patch. In contrast to [17], the mathematical model of the stress-strain state of a structure is based on the classical theory of bending of circular plates. This model makes it possible to take into account the influence of the bending of the structure, as well as consider a cut-out of an arbitrary radius.

The mathematical model of the joint suggested in the paper can be considered a generalization of the classical Goland and Reissner model of adhesively joined rods for circular symmetry [9], as well as a development of the theory of circle three-layer plates [20]. In this case, as opposed to known models [18] and [21], the upper and lower load-bearing layers have different boundary conditions.

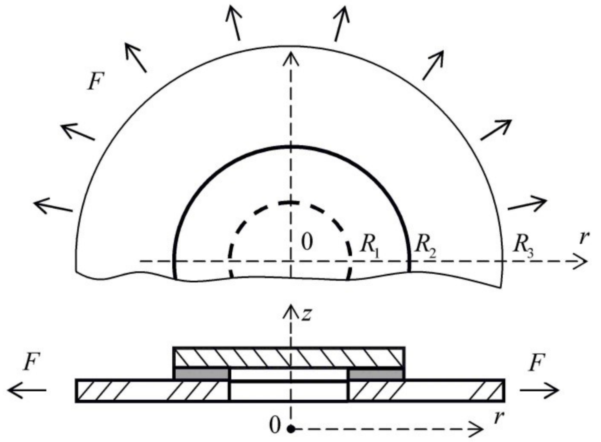


**Fig. 1** Structural diagram

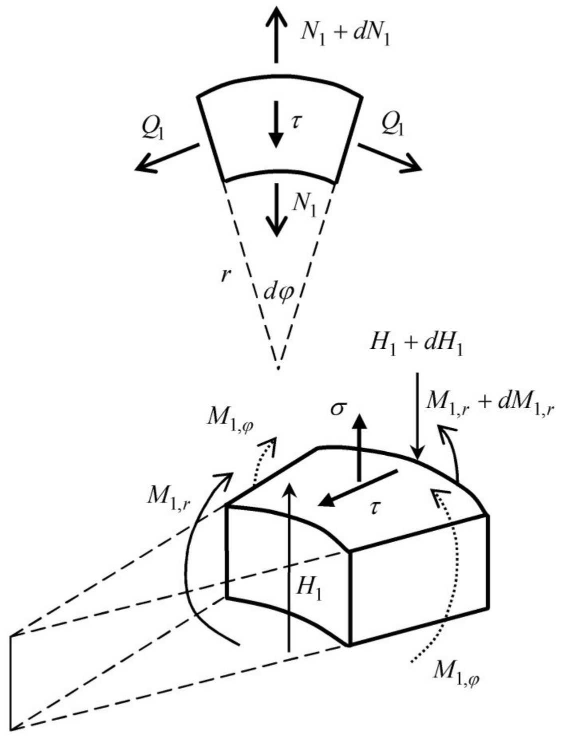


**Fig. 2** Differential element equilibrium

## 2 Materials and methods

Let us consider an adhesive joint of two circular plates shown in Fig. 1.

The radius of the hole in the main plate is $R_1$ and the radius of the patch is $R_2$. Let the main plate thickness be $\delta_1$ and that of the patch, $\delta_2$. The thickness of the adhesive layer between the plates is $\delta_0$. The main plate's outer radius is $R_3$.

The main plate is loaded with tensile forces $F$ applied along radius $R_3$. Let us consider the adhesive bond domain $r \in [R_1; R_2]$. The equilibrium of the differential element of the bottom plate is shown in Fig. 2.

Owing to axial symmetry, the forces in the load-bearing layers are independent of the angle coordinate, and tangential forces in the load-bearing layers are absent. Subscript "1" corresponds to the main plate, and subscript "2", to the circular patch within the adhesive bond domain.

The equilibrium equations of the elements of the load-bearing layers in the adhesive bond domain have the form

$$\frac{N_1 - Q_1}{r} + \frac{dN_1}{dr} - \tau = 0, \; \frac{N_2 - Q_2}{r} + \frac{dN_2}{dr} + \tau = 0\,, \tag{1}$$

$$\frac{M_{1r} - M_{1\varphi}}{r} + \frac{dM_{1r}}{dr} - H_1 + \frac{\delta_1}{2}\tau = 0\,,$$

$$\frac{M_{2r} - M_{2\varphi}}{r} + \frac{dM_{2r}}{dr} - H_2 + \frac{\delta_2}{2}\tau = 0\,, \tag{2}$$

$$-\sigma+\frac{H_1}{r}+\frac{dH_1}{dr}=0,\ \sigma+\frac{H_2}{r}+\frac{dH_2}{dr}=0\,, \tag{3}$$

where $N_k$, $Q_k$ are normal forces $k$ in the plate in the radial and circumferential directions, $k$ = 1, 2; $\tau$, $\sigma$ are tangential and normal (cleavage) stresses in the adhesive layer; $M_{kr}$ is bending moment in layer $k$ in the radial direction; $M_{k\varphi}$ is bending moment in layer $k$ in the circumferential direction; $H_k$ is shearing force in layer $k$.

The tangential and normal stresses in the adhesive layer are uniform across the thickness and are proportional to the difference of longitudinal and transverse displacements facing the adhesive layer on the sides of the plates [6, 9]:

$$\tau=P_1\left(U_1-U_2-\frac{\delta_1}{2}\frac{dW_2}{dr}-\frac{\delta_2}{2}\frac{dW_1}{dr}\right),\ \sigma=P_2\left(W_2-W_1\right), \tag{4}$$

where $P_1$ and $P_2$ is adhesive layer shear and cleavage stiffness, $P_1 = G_0/\delta_0$, $P_2=\frac{E_0}{\delta_0\left(1-\mu_0^2\right)}$, where, respectively, $E_0$, $G_0$, $\mu_0$ is elasticity modulus, shear modulus and Poisson's ratio of the adhesive; $W_k$ are transverse displacements of layer $k$; $U_k$ are longitudinal (radial) displacements of layer $k$, $k$ = 1, 2.

Hooke's law for plates takes the form

$$N_k=B_k\left(\varepsilon_{kr}+\mu_k\varepsilon_{k\varphi}\right),\ Q_k=B_k\left(\varepsilon_{k\varphi}+\mu_k\varepsilon_{kr}\right), \tag{5}$$

where $B_k = \delta_k E_k/(1-\mu_k^2)$ is membrane stiffness of the plates; $\mu_k$ is Poisson's ratio of the material of plate $k$, $k$ = 1, 2; $E_k$ is elasticity modulus of the material of plate $k$; $\varepsilon_{kr}$ and $\varepsilon_{k\varphi}$ is layer $k$ deformation in the radial and circumferential directions.

The functional relationship between displacements and deformations is

$$\varepsilon_{kr}=\frac{dU_k}{dr},\ \varepsilon_{k\varphi}=\frac{U_k}{r}\,, \tag{6}$$

The equations of axisymmetric bending of circular plates (according to the Kirchhoff-Love theory) are

$$\begin{aligned} M_{kr}&=D_k\left(\frac{d^2W_k}{dr^2}+\frac{\mu_k}{r}\frac{dW_k}{dr}\right),\\ M_{k\varphi}&=D_k\left(\mu_k\frac{d^2W_k}{dr^2}+\frac{1}{r}\frac{dW_k}{dr}\right), \end{aligned} \tag{7}$$

where $D_k = E_k\delta_k^3/(1-\mu_k^2)$is bending stiffness of the load-bearing layers.

### 2.1 Adhesive bond domain

Differentiating Eq. (4) and using Eqs. (1)–(3) and Eqs. (5)–(7), we obtain a system of equations for stresses in the adhesive layer:

$$\frac{d^3\tau}{dr^3}+\frac{2}{r}\frac{d^2\tau}{dr^2}-\left(a_1+\frac{1}{r^2}\right)\frac{d\tau}{dr}+\left(\frac{1}{r^3}-\frac{a_1}{r}\right)\tau+P_1a_2\sigma=0\,, \tag{8}$$

$$\frac{d^4\sigma}{dr^4}+\frac{2}{r}\frac{d^3\sigma}{dr^3}-\frac{1}{r^2}\frac{d^2\sigma}{dr^2}+\frac{1}{r^3}\frac{d\sigma}{dr}+a_3\sigma-P_2a_2\left(\frac{\tau}{r}+\frac{d\tau}{dr}\right)=0 \tag{9}$$

where $a_1=\frac{P_1}{B_1}+\frac{P_1}{B_2}+\frac{P_1\delta_1^2}{4D_1}+\frac{P_1\delta_2^2}{4D_2}$, $a_2=\frac{\delta_1}{2D_1}-\frac{\delta_2}{2D_2}$, and $a_3=\frac{P_2}{D_1}+\frac{P_2}{D_2}$.

We can find $\sigma$ from Eq. (8) and substitute it into Eq. (9) to obtain a linear seventh-order differential equation for tangential stresses in the adhesive. The equation obtained is not given here in view of its awkwardness. The general solution of the obtained equation is found in the form.

$$\tau=\sum_{n=1}^{6}C_nK_1\left(\lambda_nr\right)+\frac{C_7}{r}\,, \tag{10}$$

where $K_1(\lambda_n r)$ are modified Bessel functions of second kind (Macdonald's functions); $C_n$ are arbitrary constants.

Constants $\lambda_n$ are the roots of the characteristic equation

$$\lambda^6-a_1\lambda^4+a_3\lambda^2+a_2^2P_1P_2-a_1a_3=0\,.$$

Knowing $\tau$ from Eq. (8), finding the normal stresses in the adhesive layer is straightforward

$$\sigma=\sum_{n=1}^{6}C_n\alpha_nK_0\left(\lambda_nr\right), \tag{11}$$

where $\alpha_n = (\lambda_n^3 - a_1\lambda_n)/(P_1a_2)$.

Note that the stress state of overlap joining of rods or beams [6, 7, 9] is described by linear combinations of exponential functions. In the problem considered, owing to circular symmetry, the analogues of exponential functions are unbounded and non-periodic modified Bessel functions.

Using Eqs. (5) and (6), from Eq. (1) we obtain differential equations yielding analytical solutions for longitudinal displacements of plates $U_1$ and $U_2$. Substituting stresses Eq. (11) into Eq. (4), we obtain the difference of transverse displacements $W_2 - W_1$. Substituting Eq. (10) into Eq. (4) and integrating, we obtain the expression $\delta_2W_2 + \delta_1W_1$. Solving the given system of linear equations, we find the transverse displacements of the plate and patch $W_1$ and $W_2$ in the adhesive bond domain. Knowing the stresses in the adhesive layer Eqs. (10) and (11), and the longitudinal and transverse stresses, finding the linear bending moments, as well as the radial, circumferential and the shearing forces in both plates from Eqs. (5)–(7) and Eq. (2) is straightforward.

### 2.2 Domains beyond the adhesive bond

There are two domains beyond the adhesive bond, viz. the main plate ($R_2 < r < R_3$) and the patch above the cut-out ($0 < r < R_1$). Superscript "3" denotes displacements and force factors in the main plate beyond the adhesive bond. Superscript "4" denotes the displacements and force factors in the patch above the cut-out. The displacements in these areas are described by known deformations of circular plates with no shearing forces

$$\frac{d^2U_m}{dr^2}+\frac{1}{r}\frac{dU_m}{dr}-\frac{U_m}{r^2}=0 ,$$

$$\frac{d^3W_m}{dr^3}+\frac{1}{r}\frac{d^2W_m}{dr^2}-\frac{1}{r^2}\frac{dW_m}{dr}=0 ,$$

where $m = 3, 4$.

These equations have analytical solutions. The bending moments, and the radial and tangential forces are found from Eqs. (5)–(7). The shearing forces in the plates

$$H_m = D_m\frac{d}{dr}\left(\frac{d^2W_m}{dr^2}+\frac{1}{r}\frac{dW_m}{dr}\right),\ m=3,4 .$$

### 2.3 Boundary conditions

Coefficients $C_1$, $C_2$,…, $C_{12}$ and $c_1$,…, $c_4$ with $s_1$,…, $s_6$ are found from the boundary conditions and conditions of conjunction of displacements and forces on the boundaries of the domains.

In point $r = 0$, the transverse displacements should be bounded, and the longitudinal displacements should be equal to zero. Hence, it follows that $c_4 = 0$, $s_5 = 0$ .

We assume that the main plate on the outer boundary has a sliding hinged embedment and is loaded with longitudinal forces

$$W_3\left(R_3\right)=0; M_3\left(R_3\right)=0\cdot N_3\left(R_3\right)=F=\sigma_0\delta_3 .$$

This implies that tensile stresses $\sigma_0$, which form radial forces $F = \sigma_0\delta_3$, are applied over the outer perimeter of the main plate, Fig. 1.

The conjunction conditions and the conditions on the patch boundary have the form:

$$U_1\left(R_2\right)-U_3\left(R_2\right)=0;\ W_1\left(R_2\right)-W_3\left(R_2\right)=0;$$
$$\left.\frac{dW_1}{dr}\right|_{r=R_2}-\left.\frac{dW_3}{dr}\right|_{r=R_2}=0;\ M_{2r}\left(R_2\right)=0;$$
$$M_{1r}\left(R_2\right)-M_{3r}\left(R_2\right)=0;\ N_1\left(R_2\right)-N_3\left(R_2\right)=0$$
$$H_2\left(R_2\right)=0;\ N_2(R_2)=0.$$

The conjunction conditions on the inner boundary of the adhesive bond domain have the form:

$$U_2\left(R_1\right)-U_4\left(R_1\right)=0;\ W_2\left(R_1\right)-W_4\left(R_1\right)=0;$$
$$\left.\frac{dW_2}{dr}\right|_{r=R_1}-\left.\frac{dW_4}{dr}\right|_{r=R_1}=0;\ H_2\left(R_1\right)=0;$$
$$M_{2r}\left(R_1\right)-M_{4r}\left(R_1\right)=0;\ N_2\left(R_1\right)-N_4\left(R_1\right)=0;$$
$$H_1\left(R_1\right)=0; M_{1r}\left(R_1\right)=0;\ N_1\left(R_1\right)=0.$$

The given equations form a closed system of linear equations for unknown coefficients.

## 3 Results

Let us consider a structure having the following parameters: $R_1$ = 40 mm, $R_2$ = 60 mm, $R_3 = 5R_2$, $\delta_1 = \delta_3$ = 3 mm (main plate), $\delta_2 = \delta_4$ = 5 mm (patch), $\delta_0$ = 0.1 mm. We assume that the main plate and the patch are made of the same material, i.e., $E_1 = E_2$ = 70 GPa (an aluminium alloy), $\mu_1 = \mu_2$ = 0.28. The adhesive elastic parameters are $E_0$ = 0.9 GPa, $G_0$ = 0.3214 GPa (i.e., $\mu_0$ = 0.4). Since the main plate has a fairly big outer radius $R_3$, which significantly exceeds the patch radius, the boundary conditions on the plate outer boundary have no effect on the stress state of the joint.

Fig. 3 shows the graphs of normal (a) and tangential (b) stresses in the adhesive layer, Eq. (4). Solid lines show the stresses computed using the suggested analytical model (AM), and dashed lines show the stresses in the median plane of the adhesive layer computed using finite-element modelling (FEM).

The stresses are presented in dimensionless form as ratios of acting stresses and stresses $\sigma^* = F/(R_2 - R_1)$, which are forces $F$ related to the adhesive joint width. The horizontal axis coordinate $\bar{r} = (r - R_1)/(R_2 - R_1)$ is the relative distance from the hole boundary.

When creating this two-dimensional axisymmetric finite-element model, the maximum size of the final element of the adhesive layer is assigned 0.3 $\delta_0$, and the maximum size of the elements of the load-bearing layers is 0.1 $\delta_1$. The average quality of the elements is 0.8831.

The finite-element model of the joint is supplemented with chamfers of radius 2 $\delta_0$ in the corners of the load-bearing layers and flashes on the ends of the adhesive joint. These flashes are the squeezed out excess adhesive in the form of a quadrant with the radius 3 $\delta_0$. A fragment of the finite-element model in the vicinity of the adhesive joint is shown in Fig. 4 (a). A fragment of the finite-element mesh in the vicinity of the joint edge is shown in Fig. 4 (b).

The values of the stresses computed using two different models are virtually the same. Slight differences are seen near the ends of the joint. This phenomenon is well known

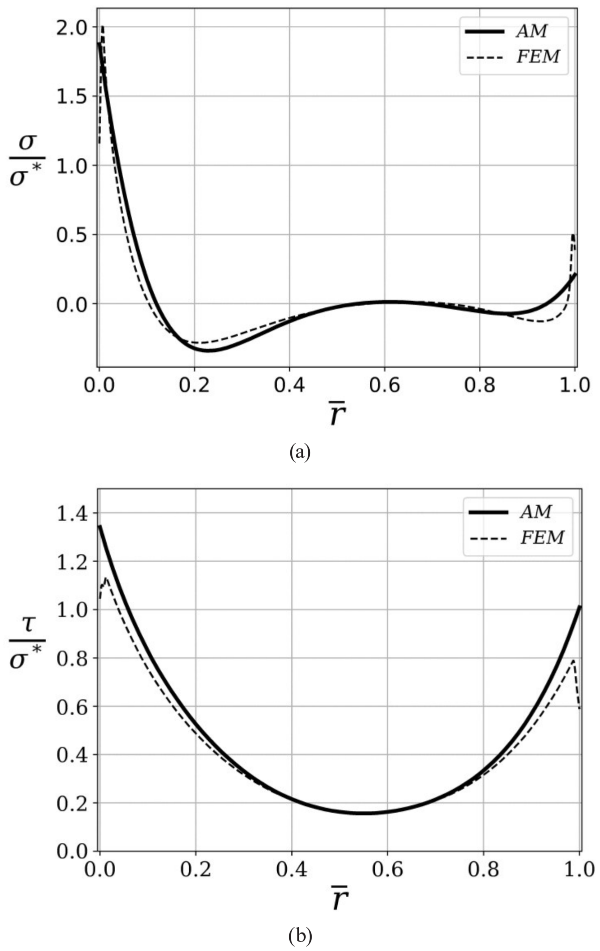


**Fig. 3** Stresses in the adhesive layer

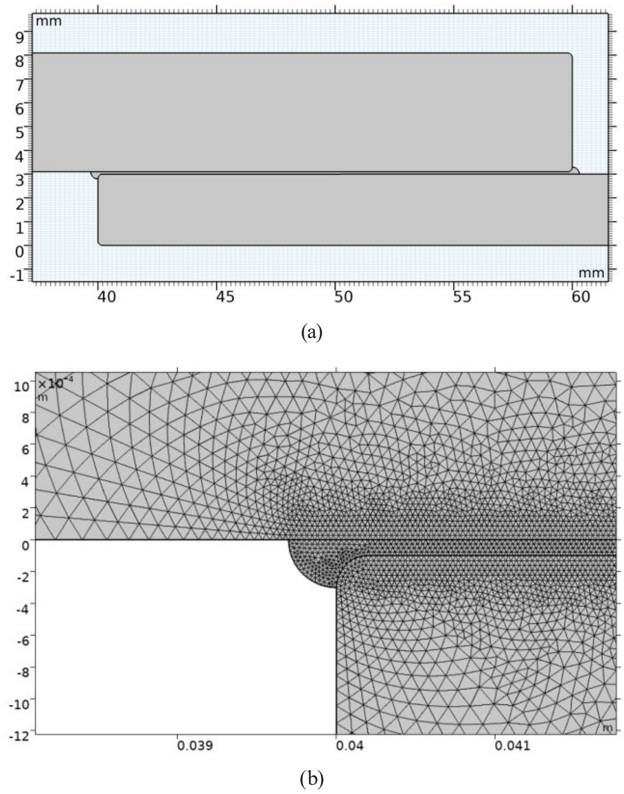


**Fig. 4** Fragments of the finite-element model

[6, 7, 9] and is due to that, according to Eq. (4), the tangential stresses are maximum on the edge of the adhesive bond domain, though this contradicts the reciprocity law for tangential stresses. This effect is significantly weakened by the presence of chamfers and excess adhesive squeezed out of the joint edge [9, 22].

## 4 Discussion

An analytical solution of the problem of determining the axisymmetric stress state of an adhesive joint of a circular plate with a cut-out reinforced with a concentric circular patch was found. The suggested mathematical model of the joint is a generalization of the classical Goland-Reissner model of the adhesive bond of two rectangular plates [9] for a circular symmetry domain. The problem is reduced to a system of differential equations for tangential and normal stresses in the adhesive layer. An important dissimilarity of the problem being considered from the classical problem of force transmission in an overlap joint of rods or rectangular plates [9, 10] is that in the given case it is impossible to ensure transmission of all forces from the main plate to the patch. This is due to that, in Eq. (1), the radial forces in the plates are offset by not only the tangential forces in the adhesive, but also by the circumferential forces in the plates.

Studies have shown that the bending of the structure contributes essentially to the stress state of the main plate in the vicinity of the cut-out. Moreover, the maximal stresses in the vicinity of the cut-out in the case of a relatively thin patch can even exceed similar stresses without a patch. To reduce the bending, it is practical to use a patch having high bending stiffness or place two similar patches on both sides of the main plate [6].

Finite-element modelling has shown that the model has an accuracy that is adequate for design tasks.

## 5 Conclusions

In this study, an axisymmetric model of the stress state of a plate that has a circular cut-out reinforced with a circular concentric patch is considered. The patch is overlapped with the main plate by means of the adhesive layer. The suggested mathematical model is based on the theory of bending of circle plates. The problem solution is obtained in the analytical form. Comparison of the calculation results with finite element modelling showed a high adequacy of the proposed model.

The offered model can be developed in the following directions.

1. The terms describing thermal strains can be added into Eq. (5) that enables us to study thermal stresses in joints.
2. The inertial terms can be added into the equations of equilibrium Eqs. (1)–(3); as a result, the stress state model makes it possible to analyze dynamic stresses in adhesive joints of plates [23, 24].
3. To describe the stress state of the adhesive layer instead of Eq. (4) a more complex two-parameter model of an elastic foundation [6, 7], can be used that it enables to take into account the boundary conditions at the outer boundary of the adhesive layer and improve the accuracy of describing the stress state of the adhesive layer at the edge of the bond area.
4. Equilibrium equations Eqs. (8) and (9) can be written for a variable thickness of the adhesive layer, which depends on the radius. A numerical solution to this problem (for example, using the finite difference method [25]) makes it possible to solve the problem of optimizing the cross section of the patch. To solve the optimization problem, we can apply a genetic algorithm.

**References**

[1] Okafor, A. C., Singh, N., Enemuoh, U. E., Rao, S. V. "Design, analysis and performance of adhesively bonded composite patch repair of cracked aluminum aircraft panels", Composite Structures, 71, pp. 258–270, 2005. https://doi.org/10.1016/j.compstruct.2005.02.023

[2] Bakuckas Jr., J. G., Chadha, R., Swindell, P., Fleming, M., Lin, J. Z., Ihn, J. B., Desai, N., Espinar-Mick, E., Freisthler, M. "Bonded repairs of composite panels representative of wing structure", In: Niepokolczycki, A., Komorowski, J. (eds.) ICAF 2019 – Structural Integrity in the Age of Additive Manufacturing, Springer, 2019, pp. 565–580. ISBN 978-3-030-21502-6 https://doi.org/10.1007/978-3-030-21503-3_45

[3] Sabelkin, V., Mall, S., Hansen, M. A., Vandawaker, R. M., Derriso, M. "Investigation into cracked aluminum plate repaired with bonded composite patch", Composite Structures, 79, pp. 55–66, 2007. https://doi.org/10.1016/j.compstruct.2005.11.028

[4] Takács, L., Szabó, F. "Experimental and Numerical Failure Analysis of Adhesive Joint of Glass Fiber Reinforced Polymer Composite", Periodica Polytechnica Mechanical Engineering, 64(1), pp. 88–95, 2020. https://doi.org/10.3311/PPme.15106

[5] Makwana, A. H., Shaikh, A. A. "The role of patch hybridization on tensile response of cracked panel repaired with hybrid composite patch: experimental and numerical investigation", The Journal of Adhesion, 97(1), pp. 53–87, 2021. https://doi.org/10.1080/00218464.2019.1629911

[6] Amidi, S., Wang, J. "An analytical model for interfacial stresses in double-lap bonded joints", The Journal of Adhesion, 95(11), pp. 1031–1055, 2019. https://doi.org/10.1080/00218464.2018.1464917

[7] Kurennov, S. S. "Refined mathematical model of the stress state of adhesive lap joint: experimental determination of the adhesive layer strength criterion", Strength of Materials, 52, pp. 779–789, 2020. https://doi.org/10.1007/s11223-020-00231-5

[8] Kurennov, S. S., Barakhov, K. P., Poliakov, A. G. "Stressed state of the axisymmetric adhesive joint of two cylindrical shells under axial tension", Materials Science Forum, 968, pp. 519–527, 2019. https://doi.org/10.4028/www.scientific.net/msf.968.519

[9] da Silva, L. F. M., das Neves, P. J. C., Adams, R. D., Spelt, J. K. "Analytical models of adhesively bonded joints. Part I: Literature survey", International Journal of Adhesion and Adhesives, 29, pp. 319–330, 2009. https://doi.org/10.1016/j.ijadhadh.2008.06.005

[10] Wong, E. H., Liu, J. "Interface and interconnection stresses in electronic assemblies – A critical review of analytical solutions", Microelectronics Reliability, 79, pp. 206–220, 2017. https://doi.org/10.1016/j.microrel.2017.03.010

[11] Rapp, P. "Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part I: general formulation", Archives of Civil and Mechanical Engineering, 10(2), pp. 81–108, 2010. https://doi.org/10.1016/s1644-9665(12)60052-1

[12] Rapp, P. "Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part II: Displacement formulation for orthotropic adherends", Archives of Civil and Mechanical Engineering, 15(2), pp. 603–619, 2015. https://doi.org/10.1016/j.acme.2014.06.004

[13] Mathias, J. D., Grédiac, M., Balandraud, X. "On the bi-directional stress distribution in rectangular bonded composite patches", International Journal of Solids and Structures, 43, pp. 6921–6947, 2006. https://doi.org/10.1016/j.ijsolstr.2006.02.016

[14] Kurennov, S. S., Barakhov, K. P. "The stressed state of the double-layer rectangular plate under shift. The simplified two-dimensional model", PNRPU Mechanics Bulletin, 3, pp. 166–174, 2019. https://doi.org/10.15593/perm.mech/2019.3.16

[15] Kurennov, S. S. "Determining stresses in an adhesive joint with a longitudinal unadhered region using a simplified two-dimensional theory", Journal of Applied Mechanics and Technical Physics, 60(4), pp. 740–747, 2019. https://doi.org/10.1134/s0021894419040199

[16] Kessentini, R., Klinkova, O., Tawfiq, I., Haddar, M. "Transient hygro-thermo-mechanical stresses analysis in multi-layers bonded structure with coupled bidirectional model", International Journal of Mechanical Sciences, 150, pp. 188–201, 2019. https://doi.org/10.1016/j.ijmecsci.2018.10.004

[17] Khan, M. A., Kumar, S. "Interfacial stresses in single-side composite patch-repairs with material tailored bondline", Mechanics of Advanced Materials and Structures, 25(4), pp. 304–318, 2017. https://doi.org/10.1080/15376494.2016.1255824

[18] Zemlyanova, A. Y. "Reinforcement of a plate weakened by multiple holes with several patches for different types of plate-patch attachment", Mathematics and Mechanics of Solids, 21(3), pp. 281–294, 2014.
https://doi.org/10.1177/1081286513519812

[19] Kurennov, S., Smetankina, N. "Stressed state of an infinite plate with a circular opening and a concentric cover plate", In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds.) Integrated Computer Technologies in Mechanical Engineering - 2020, Springer, 2021, pp. 344–354. ISBN 978-3-030-66716-0
https://doi.org/10.1007/978-3-030-66717-7_29

[20] Starovoitov, E. I., Leonenko, D. V., Tarlakovskii, D. V. "Thermo-elastic deformation of a circular sandwich plate by local loads", Mechanics of Composite Materials, 54(3), pp. 299–312, 2018.
https://doi.org/10.1007/s11029-018-9740-x

[21] Kudin, A. V., Choporov, S. V., Gomenyuk, S. I. "Axisymmetric bending of circular and annular sandwich plates with nonlinear elastic core material", Mathematical Models and Computer Simulations, 9, pp. 601–612, 2017.
https://doi.org/10.1134/S2070048217050076

[22] Frostig, Y., Thomsen, O. T., Mortensen, F. "Analysis of adhesive-bonded joints, square-end, and spew-fillet-high-order theory approach", Journal of Engineering Mechanic, 125(11), pp. 1298–1307, 1999.
https://doi.org/10.1061/(ASCE)0733-9399(1999)125:11(1298)

[23] Ugrimov, S., Smetankina, N., Kravchenko, O., Yareshchenko, V. "Analysis of laminated composites subjected to impact", In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds.) Integrated Computer Technologies in Mechanical Engineering – 2020, Springer, 2021, pp. 234–246. ISBN 978-3-030-66716-0
https://doi.org/10.1007/978-3-030-66717-7_19

[24] Baali, M., Amrane, M. N. "Effect of insertion viscoelastic damping layer with different thicknesses on the dynamic response of multi-layered beam in forced vibration", Periodica Polytechnica Mechanical Engineering, 65(1), pp. 1–9, 2021.
https://doi.org/10.3311/PPme.11110

[25] Karpov, Y. S. "Jointing of high-loaded composite structural components. Part 2. Modeling of stress-strain state", Strength of Materials, 38, pp. 481–491, 2006.
https://doi.org/10.1007/s11223-006-0067-9