COLLAPSE LOAD ANALYSIS OF PANEL STRUCTURES BY STOCHASTIC PROGRAMMING

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1. Introduction

The behaviour of buildings composed of prefabricated panels is largely influenced by displacements of the joints made on the site. The joints are the most delicate and the weakest parts of the buildings, therefore it is very important to determine the forces acting on them. They are decisive in the strength calculation, mainly in determining the collapse load of panel structures. But strength characteristics of joints made in field assembly may differ considerably from each other.

For the sake of simplicity, the method to be presented for the collapse load analysis of panel structures in plane will take the uncertainty of joint material characteristics into consideration. This method is more complicated than the usual one, but numerical results show that an error on the detriment of safety is committed by assuming given, constant material characteristic values. The uncertainties of construction are reckoned with by taking the yield points as random variables.

2. Structural aspects of the problem

Our investigation is made on rigid-body models [1]. The panels are considered as rigid bodies connected by springs capable to transmit tensile, compressive and shear forces along their edges (Fig. 1). This means that between two panel edges there are three springs: two for taking tensile or compressive forces, and one for taking the shear force.

Initial conditions:

C1: The loads acting on the structure are one-parametrical and no kinematic load is acting.

C2: The yield points of each spring are normally distributed random variables with prescribed expected value and variance.

C3: The three forces acting along one panel edge are other than independent, according to a given correlation matrix.

C4: The values of upper and lower yield points are mutually independent random variables.

The χ^2 distribution would agree better with experimental values than the normal distribution under *C2* [2]. But the computations with χ^2 distribution are more complicated and laboursome, arguing for the normal distribution.

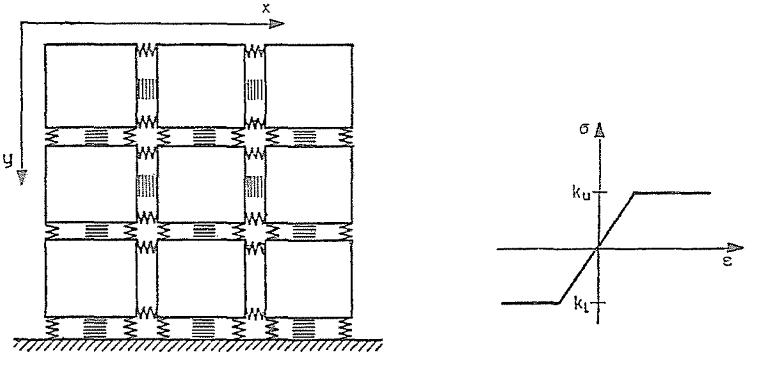

Fig. 1

The collapse load analysis problems are solved generally by mathematical programming. This method is based on the statical and kinematical theorems. Our problem will be solved on the basis of the statical theorem, stating that the statically available maximum load parameter is less than, or equal to, the collapse load parameter [3].

Let us establish, according to Fig. 2, the projection equations for axes x and y and the moment equilibrium equation of each panel. For the i-th panel:

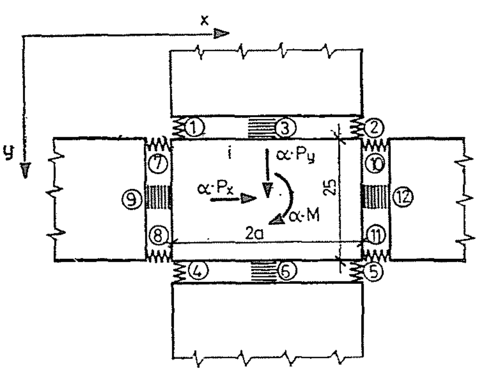

Fig. 2

$$s_7 - s_{10} + s_8 - s_{11} + s_3 - s_6 + \alpha P_x^i = 0$$
$$s_1 + s_2 - s_4 - s_5 + s_9 - s_{12} + \alpha P_y^i = 0 \tag{1}$$
$$-s_1 a + s_2 a + s_4 a - s_5 a - s_9 a + s_{12} a +$$
$$+ s_7 b - s_8 b + s_{10} b - s_{11} b + s_3 b - s_6 b + \alpha M^i = 0$$

where

s_j — force in the j-th spring;
P_x^i — projection x of the load acting on the i-th panel;
P_y^i — projection y of the load acting on the i-th panel;
M^i — moment acting on the i-th panel;
α — load parameter.

In general, it can be written for the complete panel structure:

$$\mathbb{G}^* \mathbf{s} + \alpha \mathbf{q} = \mathbb{0} \tag{2}$$

where

n — number of joints;
m — number of panels,
$\mathbb{G}\ [3 \cdot m, n]$ — geometrical matrix of the structure;
$\mathbf{s}\ [n]$ — vector of unknown forces,
$\mathbf{q}\ [3 \cdot m]$ — load vector.

For each force the yield conditions give the upper and lower limits:

$$\mathbf{k}_l < \mathbf{s} < \mathbf{k}_u \tag{3}$$

where $\mathbf{k}_l$ and $\mathbf{k}_u$ are vectors of the lower and upper limits, respectively, which are random variables.

The maximum value of load parameter

$$\alpha \rightarrow \max \tag{4}$$

is wanted.

3. Mathematic feature of the problem

It is a case of stochastic programming if at least one parameter of linear or nonlinear programming is a random variable.

The model (2), (3), (4) is that of stochastic programming with probability restriction [4].

The problem has the following form:

$$\begin{aligned} &\mathbf{G}^* \mathrm{s} + \alpha \mathbf{q} = 0 \\ &P(\mathbf{k}_l \leq \mathrm{s} \leq \mathbf{k}_n) \geq p \qquad (5) \\ &(-\alpha) \to \min. \end{aligned}$$

where p is a prescribed probability.

In order to solve the problem, α will be considered as the $(n+1)$-th unknown and the vector $\mathbf{q}$ will be attached to matrix $\mathbf{G}^*$. If $\mathbf{A}$ is the new matrix and s is the new unknown vector, the linear conditions are:

$$\mathbf{A} \cdot \hat{\mathrm{s}} = 0. \qquad (6)$$

According to *C4*, $\mathbf{k}_l$ and $\mathbf{k}_n$ are independent and the stochastic condition may be written as:

$$P(\mathbf{k}_l \leq \mathrm{s})\, P(\mathrm{s} \leq \mathbf{k}_u) \geq p. \qquad (7)$$

By definition of the distribution function, the first term in (7) is the value of the distribution function for s. Also the second term in (7) can be transformed to a distribution function. Converting (7) to a standard normal distribution:

$$\begin{aligned} &P\left\{\frac{k_l(i) - E_{k_l}(i)}{D_{k_l}(i)} \leq \frac{s(i) - E_{k_l}(i)}{D_{k_l}(i)},\ i = 1, \ldots, n\right\} \times \\ &\times P\left\{\frac{-k_u(i) + E_{k_u}(i)}{D_{k_u}(i)} \leq \frac{-s(i) + E_{k_u}(i)}{D_{k_u}(i)},\ i = 1, \ldots, n\right\} \geq p \qquad (8) \end{aligned}$$

where $E_{k_l}(i)$ and $E_{k_u}(i)$ are expected values of $k_l(i)$ and $k_u(i)$, respectively; $D_{k_l}(i)$ and $D_{k_u}(i)$ are variances of $k_l(i)$ and $k_u(i)$, respectively.

Introducing notations:

$$\xi(i) = \frac{k_l(i) - E_{k_l}(i)}{D_{k_l}(i)}; \qquad \eta(i) = \frac{-k_u(i) + E_{k_u}(i)}{D_{k_u}(i)}$$

$$\tilde{s}(i) = \frac{s(i) - E_{k_l}(i)}{D_{k_l}(i)}; \qquad \tilde{\tilde{s}}(i) = \frac{-s(i) + E_{k_u}(i)}{D_{k_u}(i)}$$

(8) will have the following form:

$$P\big(\xi(i) \leq \tilde{s}(i), i = 1, \ldots, n\big)\, P\big(\eta(i) \leq \tilde{\tilde{s}}(i), i = 1, \ldots, n\big) \geq p. \qquad (9)$$

Using the symbol of the multivariate normal distribution function $\Phi(x)$ with superscripts referring to the respective matrices of expected values, variances and correlation, (9) becomes:

$$\Phi^{(\mathbf{0}, \mathbf{1}, \mathbf{R}_l)}_{(\tilde{\mathbf{s}})} \cdot \Phi^{(\mathbf{0}, \mathbf{1}, \mathbf{R}_u)}_{(\tilde{\tilde{\mathbf{s}}})} \geq p \qquad (10)$$

where $\mathbf{R}_l$ and $\mathbf{R}_u$ are correlation matrices for $\mathbf{k}_l$ and $\mathbf{k}_u$, respectively.

Now, problem (5) has the following form:

$$\mathbf{A}\cdot\hat{\mathbf{s}}=0$$

$$\Phi^{(\mathbf{0},\,\mathbf{1},\,\mathbf{R}_l)}_{(\tilde{\mathbf{s}})}\cdot\Phi^{(\mathbf{0},\,\mathbf{1},\,\mathbf{R}_u)}_{(\tilde{\tilde{\mathbf{s}}})}\geq p \tag{11}$$

$$(-\hat{s}_{n+1})\rightarrow \text{minimum.}$$

Let us have a closer look at stochastic conditions (9).

i) Ignoring $C3$ and requiring each force to meet one stochastic condition:

$$P\big(\xi(i)\leq\tilde{s}(i)\big)\cdot P\big(\eta(i)\leq\tilde{\tilde{s}}(i)\big)\geq p(i)\quad i=1,\ldots,n \tag{12}$$

results in an error on the unsafe side. Assuming each stochastic condition to have the same p value and r conditions to be satisfied at an optimum s, according to the theorem of the product of independent random variables:

$$P\big(\xi(i)\leq\tilde{s}(i),\ i=1,\ldots,n\big)\ P\big(\eta(i)\leq\tilde{\tilde{s}}(i),\ i=1,\ldots,n\big)\geq p^r.$$

If e.g. $p=0.9$, $r=10$; $p^r<0.35$ only, even ignoring the correlation.

ii) The problem would best be solved by calculating the simultaneous probabilities of the stochastic conditions.

$$P\big(\xi(i)\leq\tilde{s}(i),i=1,\ldots,n\big)\ P\big(\eta(i)\leq\tilde{\tilde{s}}(i),i=1,\ldots,n\big)\geq p. \tag{13}$$

In this case the value of an n-dimensional normal distribution function has to be calculated. Theoretically this is possible but practically the volume of computations is prohibitive.

The arguments under i) and ii) justify assumption $C3$ on the correlation between forces. This is obvious since the joints along an edge of a panel are made simultaneously on the site.

Finally our problem (11) will have the following form:

$$\mathbf{A}\cdot\hat{\mathbf{s}}=0$$

$$\Phi^{(\mathbf{0},\,\mathbf{1},\,\mathbf{R}_{kl})}_{(\tilde{\mathbf{s}}^j(i))}\cdot\Phi^{(\mathbf{0},\,\mathbf{1},\,\mathbf{R}_{ku})}_{(\tilde{\tilde{\mathbf{s}}}^j(i))}\geq p\qquad j=1,\ldots r \tag{14}$$

$$(-\hat{s}_{n+1})\rightarrow\text{minimum}\qquad i=\begin{array}{l}3(j-1)+1\\3(j-1)+2\\3(j-1)+3\end{array}$$

where r is the number of panel edges ($r=n/3$), (14) is proved in [4] to have a solution, namely the conditions are described by either linear or concave nonlinear functions and the objective function is linear.

4. Algorithm of the solution

Our problem will be solved by SUMT method (Sequential Unconstrained Minimization Technique) elaborated by FIACCO and MCCORMICK. Denote the objective function by

$$F(x_1, \ldots, x_n)$$

the nonlinear inequality constraints by

$$G_k(x_1, \ldots, x_n) \geq 0; \ (k = 1, \ldots, n)$$

and the linear equality constraints by

$$H_k(x_1, \ldots, x_n) = 0; \ (k = n + 1, \ldots, m).$$

The basic idea of SUMT is to solve a sequence of unconstrained problems whose solutions tend to the minimum of the object function of the non-linear programming problem in the limit. The nonlinear programming problem is converted into a sequence of unconstrained problems by defining the penalty function P as follows:

$$P(x^{(k)}, r^{(k)}) = F(x^{(k)}) + \frac{1}{r^{(k)}} \sum_{i=1}^{n} H_i^2(x^{(k)}) - r^{(k)} \sum_{i=1+n}^{m} \ln G_i(x^{(k)}) \tag{15}$$

where the weighting factors $r^{(k)}$ are positive and form a monotonically decreasing sequence of values ($r^{(1)} > r^{(3)} > \ldots > r^{(i)} > \ldots > 0$). The form $H_i\,[x^{(k)}]$ was simply selected as the square sum of the respective equality constraints, so that as $r^{(k)} \to 0$, the equality constraints are ever closer satisfied.

The minimization of function (15) is initiated at an interior point $x^{(0)}$ where all the equalities and inequality constraints are satisfied. After $r^{(0)}$ is computed, $x^{(1)}$ is determined by minimizing $P(x^{(0)}, r^{(0)})$. Then $r^{(1)}$ is computed and $x^{(2)}$ is determined by minimizing $P(x^{(1)}, r^{(1)})$, and so forth. The determination of $x^{(i+1)}$ from $x^{(i)}$ involves the gradient of the conditions.

The other problem is to obtain the function values and the gradient vectors of the three-dimensional normal distribution functions.

MILTON's method [5] was applied to calculate the function values. The three-dimensional normal distribution function was transformed into product of a one-dimensional normal distribution function by a non-standard two-dimensional one, using an iterated integral. The actual value of the one-dimensional normal distribution function was calculated exactly while the actual value of the two-dimensional one was approximated by *Sympson's* rule.

The gradient of the three-dimensional distribution function was obtained by reducing the calculation to the actual value of a two-dimensional function according to the method by SZÁNTAI [6].

5. Numerical example and experiences

Let us consider the panel structure seen in Fig. 3.

The joints connecting the horizontal panel edges are type I, the others are type II.

Data are the following:

	Tensile or compressive springs				Shear springs			
	E_{k_l} [kp/cm²]	D	E_{k_u} [kp/cm²]	D	E_{k_l} [kp/cm²]	D	E_{k_u} [kp/cm²]	D
Type I	−80	1	6	1	−7	0.5	7	0.5
Type II	−50	1	20	1	−5	0.5	5	0.5

Correlation matrix between the springs along a panel edge:

$R =$	Tensile compr.	Tensile compr.	Shear
Tensile compr.	1	0.2	0.7
Tensile compr.	0.2	1	0.7
Shear	0.7	0.7	1

Under stochastic conditions the probability $p \geqslant 0.95$. This problem was solved partly with random variables and partly as a deterministic problem where the yield points assume the expected values for k_l and k_u.

The results are the following:

	s_1	s_2	s_3	s_{19}	s_{25}	s_{26}	s_{27}	α
Deterministic case	12.7	−21.3	−3.7	−76.39	−32.1	20.0	5.0	1.63
Stochastic case	10.3	−20.7	−5.2	−73.42	−35.7	18.6	4.76	1.47

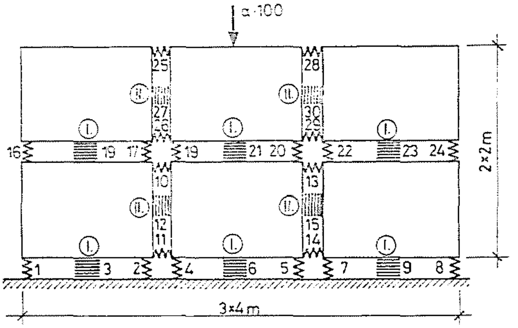

Fig. 3

The difference between deterministic and stochastic values of α is seen not to be excessive, but for the forces it may be considerable.

This method needs considerable running time. The program was run on a CDC-3300 computer, using 3064 sec CPU time. The convergence depends on the initial vector $x^{(0)}$.

Summary

Panel structures have been modelled by a system of rigid bodies connected by elastic-perfectly plastic springs. The yield points were considered as normally distributed random variables correlated to each other along a panel edge. The parameter of collapse load was determined by stochastic programming. The solution was obtained using the SUMT algorithm. The rate of convergence depends on the initial vector.

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