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Reliability Analysis of Stone Columns Improved Ground for Mitigation of Liquefaction

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Abstract

Stone columns are widely used to improve ground performance and mitigate liquefaction in weak or loose soils. However, conventional deterministic design methods fail to account for the uncertainties in soil and column properties, which can significantly affect performance under seismic loading. To address this gap, this study presents a reliability-based framework for assessing the effectiveness of stone columns in reducing liquefaction potential. The analysis considers key random variables – seismic acceleration (a_{max}), saturated unit weight (γ_{sat}), stone friction angle (φ'_c), shear wave velocity (V_s), and column diameter (D_c) – under normal and lognormal distributions. Probabilistic methods, including First Order Second Moment (FOSM), Point Estimation Method (PEM), and Monte Carlo Simulation (MCS), were applied to evaluate the reliability index (β). Results show that increasing parameter uncertainty leads to a significant reduction in β, even when the deterministic safety factor exceeds unity (≈1.50). Sensitivity analysis reveals that V_s and γ_{sat} have the greatest influence on reliability, while D_c and a_{max} exhibit minimal impact. These findings underscore the necessity of incorporating probabilistic approaches in design and provide insights for simplifying reliability assessment by reducing the number of random variables without compromising accuracy.

Keywords

reliability analysis, stone columns, liquefaction, sensitivity analysis

1 Introduction

1.1 Liquefaction hazards and mitigation practices

Soil liquefaction is a phenomenon that occurs under seismic stress, and understanding the risk of this phenomenon is becoming increasingly important, even in the most modest projects. Liquefaction leads to the loss of strength in water-saturated sandy materials and is caused by an increase in pore pressure generated by cyclic deformations. Liquefaction most commonly occurs in saturated soils; however, research has shown that unsaturated soils with high degrees of saturation (above ~70%) can also liquefy under seismic loading [1]. Additionally, the grain size of the material plays a significant role, with soils prone to liquefaction under cyclic loading typically being silty to sandy and having a narrow grain size distribution. A schematic of the liquefaction process is presented in Fig. 1, adapted from Akar et al. [2].

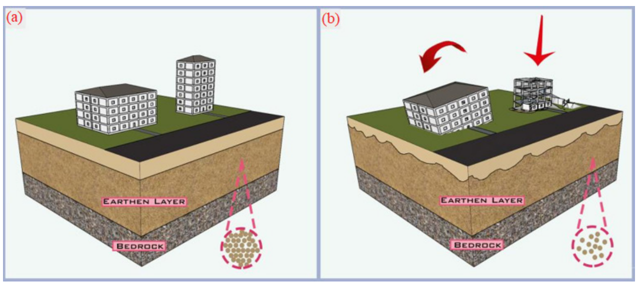

Fig. 1 Schematic of liquefaction mechanics: (a) Unliquefied soil; (b) Liquefied soil [2]

The devastating impacts of liquefaction have been repeatedly observed during past earthquakes. One of the most recent and severe examples occurred during the February 6, 2023, Kahramanmaraş earthquake sequence in Türkiye. This event underscored the widespread con-

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sequences of liquefaction, including structural collapses, infrastructure failure, and permanent ground deformation. Field investigations across affected provinces – such as Hatay, Adıyaman, and Kahramanmaraş – revealed pervasive sand boils, ground fissures, lateral spreading, and bearing capacity failures [2–4].

At two petrochemical facilities in Hatay, Sahin and Onder Cetin [3] documented severe liquefaction-induced ground deformations that disrupted operations and exceeded predicted values from conventional semi-empirical methods. In Gölbaşı (Adıyaman), liquefaction caused the tilting and collapse of reinforced concrete and masonry buildings – linked to both poor soil – structure interaction and construction deficiencies [2, 4]. Surprisingly, fine-grained soils with moderate to high plasticity also exhibited liquefaction behavior, including the ejection of clayey materials, thereby challenging long-standing assumptions about susceptibility thresholds [4]. Further assessments highlighted systemic failures in design and workmanship, as well as the limitations of traditional liquefaction prediction models that exclude the effects of lateral spreading and clay-rich soils [5, 6]. These findings underscore the urgent need to revise existing liquefaction assessment methodologies and adopt more reliable hazard mitigation approaches. Representative images of liquefaction-induced damage are shown in Figs. 2 and 3, illustrating excessive settlement, loss of soil bearing capacity, soil ejecta, and lateral spreading. These visuals, drawn from recent field case studies, demonstrate the practical consequences of liquefaction and the necessity for improved mitigation solutions.

To mitigate liquefaction risks, several ground improvement techniques have been developed and applied, depending on soil conditions, project constraints, and seismic hazard levels. These include soil densification (e.g., vibro-compaction, dynamic compaction), drainage enhancement (e.g., gravel or prefabricated vertical drains), grouting methods, geosynthetic reinforcement, deep soil mixing, and induced partial saturation. These approaches aim either to improve the soil's mechanical behavior or to reduce excess pore pressure development during seismic events. Among these techniques, stone columns are widely used due to their ability to increase shear strength, improve drainage, and densify the surrounding soil. Their relative cost-effectiveness, ease of installation, and dual action (strengthening and drainage) make them one of the most effective countermeasures against liquefaction in silty and sandy soils.

Fig. 2 Examples of structural damage caused by settlement and loss of bearing capacity due to seismic liquefaction in Gölbaşı, Türkiye [2, 5, 6]

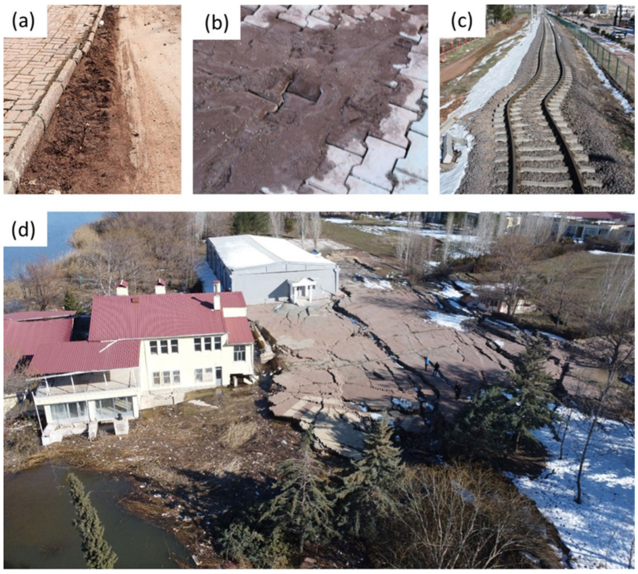


Fig. 3 Sand ejecta and lateral spreading resulting from seismic liquefaction [6]

1.2 Literature review on stone columns for liquefaction mitigation

Over the years, a broad range of field studies have confirmed the effectiveness of stone columns in real-world seismic settings. Early full-scale field tests demonstrated that stone columns improve shear strength and densify loose sands, thus significantly reducing the risk of liquefaction [7–8]. In marine and port environments, stone columns have been successfully used to treat liquefiable silty and sandy layers and to enhance seismic safety [9]. Several applied case studies continue to support their growing use in diverse geotechnical projects. Complementing field observations, experimental investigations – including shaking table tests and laboratory model studies – have provided controlled evidence of liquefaction mitigation by stone columns. These studies have demonstrated that stone columns improve excess pore pressure dissipation and reduce ground deformations under seismic loading. Notable contributions include laboratory and physical model tests [10–12], which show that both ordinary and encased stone columns can significantly enhance liquefaction resistance in sandy and silty soils. Shenthan et al. [13] also introduced composite stone columns as an effective countermeasure in non-plastic silts, while Pal and Deb [14] analyzed the impact of column stiffness on drainage performance during liquefaction. A substantial number of numerical modeling studies have also explored the performance of stone columns under various seismic scenarios. These simulations have provided insights into the effects of column spacing, area replacement ratio, permeability, encasement, and soil stratification [15–19]. In particular, numerical analyses have emphasized the role of geosynthetic encasement and filtering techniques in enhancing column efficiency during liquefaction events.

1.3 Influence of local soil conditions on liquefaction behavior

The susceptibility and severity of seismic soil liquefaction are strongly governed by local geotechnical conditions, including stratigraphy, saturation level, grain size distribution, plasticity, and stiffness. Variations in these properties can significantly influence pore pressure generation, loss of shear strength, and ground deformations under seismic loading. Accurate evaluation of liquefaction hazards therefore requires a clear understanding of site-specific soil behavior. Recent studies have emphasized the importance of integrating detailed in-situ data with empirical and probabilistic approaches to assess the influence of local soil conditions. Eyisüren et al. [20] conducted a comprehensive investigation in the Çanakkale region, combining Standard Penetration Test (SPT) and shear wave velocity (V_s) measurements to develop liquefaction risk maps. Their work demonstrated that Vs-based parameters can reduce uncertainties related to SPT energy corrections and improve liquefaction prediction in urban environments with variable soil profiles. In another study, Işık et al. [21] examined the impact of geotechnical conditions on the seismic performance of reinforced concrete structures during the 2023 Kahramanmaraş earthquakes. The authors reported that structures located on thick sedimentary layers and soft soils experienced disproportionate damage, including tilting, foundation failure, and lateral spreading. Their findings underscore the importance of realistic soil–structure interaction modeling and the consideration of liquefaction potential in site-specific seismic design. To enhance predictive reliability, Tuna [22] applied a performance-based probabilistic framework in Iskenderun, integrating post-earthquake observations with ground motion and geotechnical data. The study introduced fragility functions linking the Liquefaction Potential Index (LPI) with the Damage Severity Index (DSI), allowing for quantification of liquefaction-related risks across different soil conditions. On the mitigation side, Shen et al. [23] investigated the use of fiber reinforcement to reduce liquefaction-induced deformations in tunnel environments. Shake table experiments showed that carbon fiber inclusion improved soil stiffness, reduced excess pore water pressure, and minimized tunnel uplift, confirming the potential of localized soil improvement techniques to enhance seismic performance in liquefiable ground. Collectively, these studies confirm that local soil conditions play a critical role in the occurrence, severity, and consequences of seismic liquefaction. Incorporating site-specific geotechnical data into both hazard assessment and mitigation design is essential for achieving resilient and reliable seismic performance.

1.4 Limitations of deterministic approaches and research motivation

Most studies on the behavior of stone columns rely on deterministic methods, which do not fully reflect the inherent variability of soil properties. Given the natural heterogeneity of soils, variations in geotechnical characteristics can significantly influence the liquefaction response of stone column-improved ground. Traditional factor of safety approaches may offer conservative design margins but fail to quantify uncertainty or the probability of liquefaction occurrence. In response to this limitation, several researchers have employed reliability-based methods to better account for geotechnical variability. Elshazly et al. [24] investigated settlement prediction using back-analysis and proposed correction factors based on field-calibrated data. Marandi et al. [25] explored the uncertainty in embankment safety factors using fuzzy α-cut techniques, emphasizing the influence of groundwater and embankment height. Pham et al. [26] applied probabilistic methods to evaluate the internal stability of geosynthetic-reinforced column-supported embankments, demonstrating that bending failure governs reliability under seismic loads. Similarly, Guechi and Bordjiba [27] applied multiple reliability techniques – First Order

Second Moment (FOSM), Point Estimate Method (PEM), and Monte Carlo Simulations (MCSs) – to assess the bearing capacity of stone columns and identified key geotechnical parameters that should be treated as random variables in reliability-based design.

Extending this line of research, the present study adopts a reliability-based approach to evaluate the liquefaction performance of ground improved with stone columns. Three probabilistic methods – FOSM, PEM, and MCS – are employed to determine the reliability index (β) for liquefaction mitigation and to quantify the impact of geotechnical uncertainties on system performance. Numerical analyses are performed using MATLAB [28]. The study addresses four key objectives:

1. Quantify the effects of parameter uncertainty on stone column reliability;
2. Perform a comparative evaluation of the selected reliability methods;
3. Determine appropriate probability distributions for input parameters;
4. Conduct sensitivity analysis to identify the most influential variables and reduce the number of random inputs required for robust reliability assessment.

These contributions collectively advance the development of more resilient, reliability-based design methodologies for liquefaction mitigation.

2 Stone columns for mitigation of liquefaction

Stone columns are utilized to dissipate pore pressures induced by seismic action in the soil. In addition, their rigidities contribute to reducing the intensity of shear forces resulting from seismic action on the ground. Fig. 4 [29] illustrates the principle of reducing shear stresses on soils through the presence of stone columns. The seismic shear stress taken up by the soil (τ_s) is represented as the difference between the seismic shear stress (τ) and the seismic shear stress absorbed by the stone columns (τ_c), i.e., $\tau_s = \tau - \tau_c$.

Several approaches have been developed to assess liquefaction potential. The most widely used method is

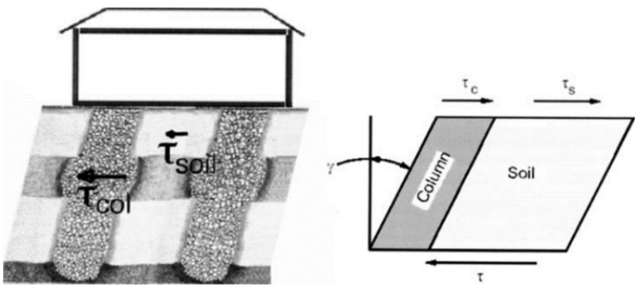


Fig. 4 Basic principle: reduction of cyclic shear stresses in the soil [29]

based on comparing cyclic stresses induced by an earthquake with the cyclic resistance of the ground. The Seed and Idriss [30] method for assessing liquefaction risk is extensively applied in practice. It provides the Cyclic Stress Ratio (CSR), which is the average cyclic shear force in a layer (τ_{avg}) normalized in relation to the effective stress $\left(\sigma'_{v0}\right)$ of natural soil subjected to an earthquake of magnitude $M_w = 7.5$, as given by the Eq. (1):

$$\mathrm{CSR} = \frac{\tau_h}{\sigma'_{v0}} = 0.65 \frac{\sigma_{v0}}{\sigma'_{v0}} \frac{a_{\max}}{g} r_d, \tag{1}$$

where $a_{\max}$ is the maximum amplitude of the horizontal acceleration at the ground surface; σ_{v0} and σ'_{v0} are the total and effective vertical stresses of the underlying soils; r_d is the depth-dependent stress reduction coefficient.

The incorporation of stone columns in soils likely to liquefy reduces the CSR by applying a stress reduction factor (SR) derived from the Priebe [31] method. This method provides a theoretical framework for estimating shear stress reduction in stone column-improved ground by modeling the composite soil-column system as a unit cell under the following key assumptions:

1. Equal vertical strain in columns and surrounding soil;
2. Radial drainage conditions;
3. Elastic–perfectly plastic behavior of the column material;
4. Negligible installation effects on long-term performance.

The method introduces two fundamental parameters: the area replacement ratio A_c/A (column cross-section A_c divided by the total treated area A), and the active earth pressure coefficient of the column material $K_{ac} = \tan^2(45° - \varphi_c/2,)$ where φ_c is the column material friction angle. From these, the stress concentration ratio n_0 is obtained as $n_0 = \sigma_c/\sigma_s$, where σ_c and σ_s are the average vertical stresses in the column and soil, respectively. The shear SR is then expressed as:

$$\mathrm{SR} = \frac{1}{n_0} = 1 + \frac{A_c}{A}\left(\frac{1}{K_{ac}\left(1 - A_c/A\right)} - 1\right). \tag{2}$$

In liquefaction assessment, the reduced CSR in improved ground is obtained by applying SR to the natural CSR: $\mathrm{CSR}_{\mathrm{improved}} = \mathrm{CSR}_{\mathrm{natural}} \times \mathrm{SR}$.

In a soil treated by the technique of stone columns against the risk of liquefaction, the CSR is a function of the seismic loading and of the columns themselves, while the ratio of the resistance to cyclic shear (Cyclic

Resistance Ratio (CRR)) represents the resistance to soil liquefaction. This resistance for an earthquake of amplitude of 7.5 ($CRR_{7.5}$) is a function of a normalized resistance parameter from an in-situ test ($(N_1)_{60}$, q_{c1}, V_{s1}).

The use of V_s as an index of resistance to liquefaction is based on the fact that the resistance of a soil to liquefaction and V_s are influenced by the same factors: the voids index, the stresses state in the soil, the history of stresses, cementation and geological age.

The calculation of the CRR by the method of Andrus and Stokoe [32] is given as a function of the corrected V_s in the ground (V_{s1}):

$$\mathrm{CRR} = \left\{ a\left[\frac{V_{s1}}{100}\right]^2 + b\left[\frac{1}{V_{s1}^* - V_{s1}} - \frac{1}{V_{s1}^*}\right]\right\}\mathrm{MSF}, \tag{3}$$

where a and b are adjustment parameters, equal to 0.03 and 0.9 respectively; V_{s1} is the shear rate corrected with respect to the effective vertical stress, equal to: $V_{s1} = V_s\left(P_a/\sigma'_{v0}\right)^{0.25}$; Magnitude Scaling Factor (MSF) is the scaling factor, equal to $(M_w/7.5)^{-2.56}$; V_{s1}^* is the limiting value of the V_s, marking the start of liquefaction in sandy soils, it was estimated by: V_{s1}^* = 215 m/s for $FC(\%) \leq 5$, V_{s1}^* = 215 − 0.5(FC − 5) m/s for 5 < $FC(\%)$ ≤ 35 and V_{s1}^* = 200 m/s for FC(%) ≥ 35.

Recent numerical studies have critically re-examined the assumptions underlying shear stress reduction in stone column design, as illustrated in Fig. 4. Rayamajhi et al. [33], using three-dimensional finite element analysis, demonstrated that the assumption of shear strain compatibility between stone columns and surrounding soil significantly overestimates the stress transferred to the columns, with actual shear stress reduction being much lower than predicted by simplified models. Similarly, Demir and Özener [34], through nonlinear finite element modeling validated by centrifuge testing, found that high-modulus columns did not substantially reduce seismic shear stresses compared to unimproved soil and that pure shear strain compatibility was not achieved during shaking. In a subsequent parametric study, Demir and Özener [35] showed that strain compatibility assumptions often lead to an overestimation of liquefaction mitigation effectiveness, emphasizing the role of strain incompatibility in actual performance. Finally, based on a full-scale field case in Christchurch, New Zealand, Demir and Özener [36] reported that stress reduction under strain incompatibility conditions was significantly lower than predicted by current design approaches relying on strain compatibility. Despite these findings, which suggest caution when applying simplified shear redistribution models, the present study retains the Baez [29] framework for three reasons:

1. To maintain methodological consistency with current design codes;
2. To enable direct comparison with existing reliability studies;
3. To provide a conservative baseline for probabilistic analysis.

3 Reliability analysis methods

3.1 First Order Second Moment (FOSM) method

The FOSM method, proposed by Cornell [37], approximates the limit state function $g(\boldsymbol{X})$ using a first-order Taylor series expansion about the mean values of the basic random variables. This allows the expected value μ_g and standard deviation σ_g of $g(\boldsymbol{X})$ to be calculated directly from the means and standard deviations of the input variables. The β is then defined as the ratio of these first two statistical moments [38].

In the general case, resistance R and load effect S may be dependent and expressed as functions of several variables with arbitrary probability distributions, and $g(\boldsymbol{X})$ can take any mathematical form:

$$g(\boldsymbol{X}) = g(x_1, x_2, \ldots, x_n). \tag{4}$$

The Taylor series expansion around the mean vector of the random variables is:

$$\begin{aligned} g(x_1, x_2, \ldots, x_n) &\approx g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}) \\ &+ \sum\nolimits_{i=1}^{n} (x_i - \mu_{x_i}) \partial g/\partial x_i. \end{aligned} \tag{5}$$

Since the expected value of the second term is zero by definition of the mean, neglecting higher-order terms yields:

$$\mu_g \approx g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}). \tag{6}$$

Similarly, the variance of $g(\boldsymbol{X})$ is obtained as:

$$Var[g] = \sigma_g^2 = E\left[(g - \mu_g)^2\right], \tag{7}$$

$$\begin{aligned} \sigma_g^2 &= \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} \rho X_i X_j \sigma_{X_i} \sigma_{X_j} \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} \\ &= \sum\nolimits_{i=1}^{n} \sigma_{x_i}^2 \left(\frac{\partial g}{\partial x_i}\right)^2 + \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} \mathrm{cov}(X_i, X_j) \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j}, \end{aligned} \tag{8}$$

where $\rho X_i X_j$ is the correlation coefficient and $\mathrm{cov}(X_i, X_j)$ the covariance between X_i and X_j. For uncorrelated variables, this simplifies to:

$$\sigma_g^2 = E\left[\left(\sum_{i=1}^{n}\left(x_i - \mu_{x_i}\right)\frac{\partial g}{\partial x_i}\right)^2\right]. \tag{9}$$

The partial derivatives in these equations are evaluated at the mean values of all parameters. While the FOSM method is computationally efficient, neglecting higher-order terms may introduce significant error for nonlinear or complex limit state functions. Additionally, it can yield different results for mechanically equivalent formulations and does not explicitly account for the full probability distributions of the variables. Therefore, it is most accurate when the input variables are normally distributed and $g(\boldsymbol{X})$ is approximately linear [38].

3.2 Point Estimate Method (PEM)

The PEM, introduced by Rosenblueth [39], provides an efficient numerical approach for estimating the statistical moments of a performance function by evaluating it at a limited set of discrete points. The original method requires $2N$ evaluations for N uncertain parameters, which can be computationally demanding when the number of variables is large. Later adaptations reduced the required evaluations to approximately $2N$, although these introduce additional complexities [40].

Let $\boldsymbol{X}$ be a continuous random variable with probability density function (PDF) $f_X(x)$, and let $Y = g(\boldsymbol{X})$ be a deterministic performance function representing, for example, soil properties, geometric parameters, or loading conditions. In PEM, the continuous variable $\boldsymbol{X}$ is replaced with a discrete variable whose probability mass function (PMF), $P_X(x)$, matches the first m moments of the PDF $f_X(x)$. The first moment of $f_X(x)$ is:

$$\mu_X = \int x f\left(x\right) dx. \tag{10}$$

The m-th central moment is:

$$\mu_{Xm} = \int \left(x - \mu_X\right)^m f\left(x\right) dx. \tag{11}$$

For the discrete PMF, the corresponding moment is:

$$\mu_{Xm} = \int \left(x - \mu_X\right)^m P_X\left(x\right). \tag{12}$$

Using Rosenblueth's notation (Fig. 5 [39]), the expected value of Y_m is given by:

$$E\left[Y^m\right] = P_+ y_+^m + P_- y_-^m, \tag{13}$$

where $Y = g(\boldsymbol{X})$ is the deterministic function, y_+ and y_- are the values of Y at points $x_+ > \mu(x)$ and $x_- < \mu(x)$, and P_+ and P_- are their associated weights.

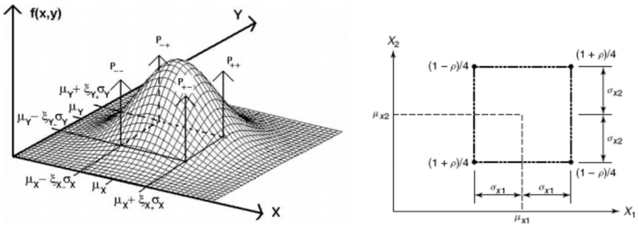


Fig. 5 PEM for two random variables [39]

For multiple correlated variables, the generalized probability weighting is:

$$P_{S_1, S_2, \ldots S_n} = \frac{1}{2^n}\left[1 + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(S_i\right)\left(S_j\right)\rho_{ij}\right], \tag{14}$$

and the m-th moment is approximated as:

$$E\left[Y^m\right] \approx \sum P_i\left(y_i\right)^m, \tag{15}$$

where S_i, S_j = +1 for values above the mean and −1 otherwise.

An alternative approach, the Independent Perturbations Method proposed by Bolle [41], uses a three-point discretization of the probability distribution of each random variable, combined with a Taylor series expansion and transformation into uncorrelated principal variables. This method is particularly efficient when the number of random variables is large. In PEM and related methods, the probability of failure (p_f) is usually computed from the β, whereas the MCS approach allows for its direct estimation.

3.3 Monte Carlo Simulation (MCS) approach

The MCS method estimates the p_f or the β by repeatedly generating random sets of values for the uncertain parameters and evaluating the performance (limit state) function for each set. The method is conceptually straightforward, relying on the law of large numbers, but achieving sufficient accuracy often requires a large number of simulations, especially when p_f is small [38, 40, 42].

The basic steps of MCS are:

1. Problem definition: identify all random variables;
2. Characterization: define their PDFs and parameters;
3. Sampling: generate random values for all variables (using random number generators from software such as MATLAB [28]);
4. Evaluation: compute the deterministic response for each realization;
5. Post-processing: calculate p_f and/or β from the set of results.

To obtain a reliable estimate of p_f, the number of simulations N should generally satisfy:

$$N = \frac{C}{p_f}, \tag{16}$$

where C is a confidence level parameter, often taken as $C = 1000$. For rare events (very small p_f), the required N may become prohibitively large. Variance reduction techniques are therefore recommended to improve efficiency and reduce computational time.

While MCS is one of the most flexible and broadly applicable reliability analysis methods – capable of handling any form of limit state function and variable distribution – it does not inherently identify the relative influence of each variable on the outcome. Moreover, its main drawback is the potentially high computational cost required for accurate probability estimates [38].

4 Reliability analysis of liquefaction

In the reliability analysis of stone columns against the risk of liquefaction in seismic zones, we are interested in presenting the soil conditions as well as the parameters linked to the columns and soils to be improved, among these parameters, some are considered random; others are deterministic and are called limit state function parameters.

The three probabilistic methods used are FOSM, PEM, and MCS. Two types of random variable distributions were considered: normal and lognormal. The variation of the β with the coefficient of variation (COV) of a given parameter was obtained by changing the standard deviation of that parameter while keeping its mean constant. During this process, the COVs of all other parameters were held fixed at baseline values of 2%, 5%, or 10%. The 2% value represents a very low uncertainty scenario typical of well-controlled geotechnical properties, 5% represents moderate uncertainty, and 10% represents high uncertainty. This approach isolates the effect of each parameter's variability on β while maintaining a constant baseline variability in the remaining parameters.

This study examines a soil layer susceptible to liquefaction under seismic excitation and evaluates the effectiveness of stone columns in mitigating this risk. Stone columns installed in the surface layer serve two primary purposes:

1. Dissipating excess pore water pressures induced by seismic loading;
2. Improving soil density, thereby increasing resistance to liquefaction.

The deterministic analysis of this case was previously conducted by Al-Homoud and Degen [9], whereas the present work focuses on a probabilistic assessment.

The analysis considers the key parameters that most significantly influence liquefaction potential and stone column performance. Table 1 summarizes the characteristics of the soil–stone column system adopted in this study. These parameters were selected because they directly affect the CSR and CRR, which together define the performance function in the reliability framework.

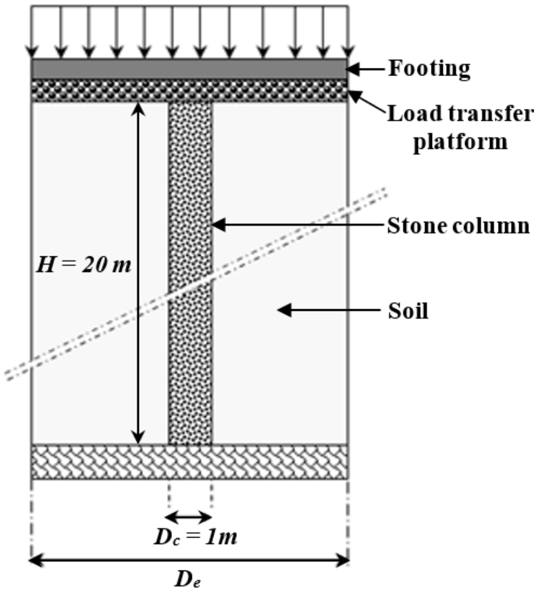
The five random variables – seismic acceleration ($a_{\max}$), saturated unit weight (γ_{sat}), column diameter (D_c), stone friction angle (φ_c'), and Vs – are widely recognized in the literature as the most influential factors governing liquefaction and improvement effectiveness [9, 30, 31]. Their values were primarily adopted from the case study of Al-Homoud and Degen [9], which provides experimentally validated parameters widely used in liquefaction mitigation design to ensure consistency with typical field conditions. Specifically, the φ_c' and D_c correspond to standard design specifications for stone columns in silty sand, the V_s and γ_{sat} reflect commonly observed values in liquefiable soil profiles, and the $a_{\max}$ was selected to represent strong earthquake loading scenarios typical of the design case.

A schematic representation of the stone column layout and dimensions is illustrated in Fig. 6. The configuration consists of columns with a diameter of 1 m, installed in an equilateral triangular grid with 1.6 m spacing, extending to a depth of 20 m within the liquefiable silty sand layer. The crushed gravel aggregate used in the columns was assigned a friction angle of 40°, consistent with commonly reported values for stone column materials.

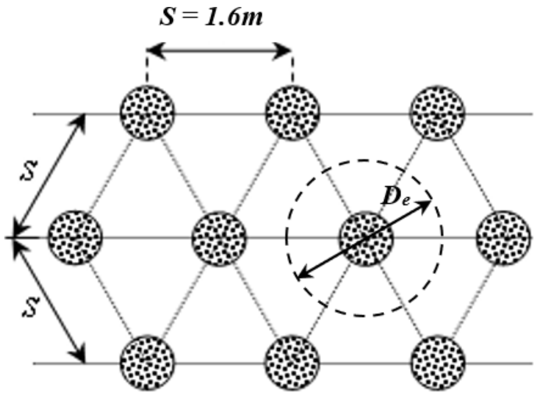
In traditional design methods, the liquefaction resistance provided by stone columns is often evaluated using deterministic approaches, where a safety factor (*Fs*) is employed to reduce uncertainties associated with the evaluation of parameters. This safety factor is then used to formulate

Table 1 Characteristics of soil-stone column system variables [9]

Parameter	Symbol	Variable	Mean
Seismic acceleration	$a_{\max}$	Random	0.25 g
Saturated unit weight	γ_{sat}	Random	21 kN/m^3
Column diameter	D_c	Random	1 m
Stone friction angle	φ_c'	Random	40°
Shear wave velocity	V_s	Random	200 m/s
Percentage of fines	F_c	Determinist	15%
Magnitude of the earthquake	M_w	Determinist	7.5
Spacing	S	Determinist	1.6 m



(a)



(b)

Fig. 6 Schematic representation and dimensions of the stone columns used in the analysis: (a) Cross-sectional view; (b) Plan view

performance functions in reliability analysis to calculate the β, as defined by Hasofer and Lind [43]. The performance function (limit state) is typically defined as:

$$g(\boldsymbol{X}) = Fs - 1, \tag{17}$$

where $\boldsymbol{X}$ denotes the set of random input parameters. The case of $g(\boldsymbol{X}) = 0$ separated the stable ($g(\boldsymbol{X}) > 0$) and unstable ($g(\boldsymbol{X}) < 0$) states, or the safe and failure regions.

In this paper, the performance function in reliability analysis for the case study is expressed by the following relationship:

$$\begin{aligned} & g\left(V_s, \gamma_{sat}, a_{\max}, Fc, \varphi_c, D_c, M_w\right) \\ & = \mathrm{CRR}\left(V_s, \gamma_{sat}, Fc\right) - \mathrm{CSR}\left(a_{\max}, \varphi_c, D_c, \gamma_{sat}, M_w\right), \end{aligned} \tag{18}$$

where the CRR is determined using Eq. (3); and the CSR, to which a stress ratio-dependent reduction is applied, is calculated by the method of Seed and Idriss [30].

The calculation of deterministic safety factors is made by comparing the CRR against the CSR. This comparison yields values of 1.50 for both single isolated columns and groups of columns, which are typically accepted according to the recommendations in Berthelot et al. [44].

The uncertainties associated with designing parameters are not incorporated in a systematic manner, and the safety factor is not considered an ideal indicator for reliability assessment, as it is determined based on subjective judgment. The p_f is considered an objective measure of reliability and is expressed by the following integral:

$$p_f = P\left[g(\boldsymbol{X}) < 0\right] = \int_{g(\boldsymbol{X})<0} f_X(\boldsymbol{X})\, d\boldsymbol{X}. \tag{19}$$

The β is a measure of an engineering system's reliability that accounts for both the mechanics of the problem and the uncertainties in the input variables [40, 45–47]. It is expressed in terms of the failure probability through the standard normal distribution function as follows:

$$\beta = -\Phi^{-1}\left(p_f\right). \tag{20}$$

In Eq. (19), $f_X(\boldsymbol{X})$ represents the joint PDF of the vector of continuous random variables ($\boldsymbol{X}$), and (Φ^{-1}) is the inverse of the standard normal cumulative distribution function. These two reliability measures ensure a consistent level of safety while incorporating the uncertainties associated with design parameters within a rational probabilistic framework.

One of the subjects of extensive research in reliability analysis is calculating the pf. However, this task can be challenging in practice, leading to the development of approximate techniques.

FOSM, Point Estimation Method (PEM), and MCSs are effective methods in geotechnical engineering that are now being utilized and evaluated. In this study, the deterministic model of stone columns for improving ground to mitigate liquefaction was analyzed using probabilistic methods, including FOSM, PEM, and MCs. Calculations for the PEM method were conducted using an Excel spreadsheet, while MATLAB software [28] was employed for programming FOSM and MCs. FOSM and PEM are classified as Level II reliability analysis methods – fully probabilistic with approximations – where uncertainties are represented by the first two statistical moments (mean and standard deviation) of each variable, and PDFs are often approximated by equivalent normal distributions. These Level II methods are particularly effective for small failure probabilities but less suited for large probability domains. In contrast, MCs represent a Level III method – fully probabilistic – using complete probability distributions of the input variables and repeated random sampling to directly estimate the p_f. While more computationally intensive, Level III methods provide a more accurate and distribution-sensitive evaluation of system reliability.

In this study, all reliability analyses – whether based on Level II methods (FOSM, PEM) or the Level III method (MCS) – were carried out within a unified computational framework. The five principal random variables ($a_{\max}$, γ_{sat}, D_c, φ'_c, V_s) were assigned mean values, coefficients of variation (COV), and probability distribution types (normal or lognormal). The CSR was computed using Eq. (1) with the shear SR from Eq. (2), and the CRR was calculated using Eq. (3), together defining the performance function $g(\boldsymbol{X})$ as expressed in Eq. (18). For MCS, large random samples were generated from the assigned distributions, $g(\boldsymbol{X})$ was evaluated for each sample, and the p_f and β were estimated directly. For FOSM, the mean of $g(\boldsymbol{X})$ was calculated using the mean values of the variables, while its standard deviation was obtained *via* a first-order Taylor series expansion involving partial derivatives with respect to each variable; β was then computed as the ratio of the mean to the standard deviation. For PEM, $g(\boldsymbol{X})$ was evaluated at selected weighted points to estimate its first two moments, from which β was computed. This unified framework ensures that differences between methods arise exclusively from their probabilistic treatment, not from the deterministic model.

5 Results and discussions

The influence of the COV of different design parameters on the β for liquefaction mitigation using stone columns was analyzed using three probabilistic methods: MCS, PEM, and FOSM. Figs. 7 (a) to (e) and Fig. 8 (a) to (e) present the results for normal and lognormal distributions, respectively, with all non-varying parameters held at a baseline COV of 2% (very low uncertainty). Overall, the results show that increasing the COV of any parameter leads to a continuous decrease in β, which corresponds to an increased p_f due to uncertainty propagation in the mechanical model.

Among the five parameters, V_s exhibits the greatest sensitivity, with β decreasing significantly as its COV increases and declining below 1 when the COV exceeds 10%, even though the average safety factor is greater than unity (≈1.50). The γ_{sat} also strongly influences β, whereas $a_{\max}$, φ'_c, and D_c show comparatively lower sensitivity, maintaining β values above 1 at higher COVs. Furthermore, Figs. 7 (a) to (e) and Fig. 8 (a) to (e) show

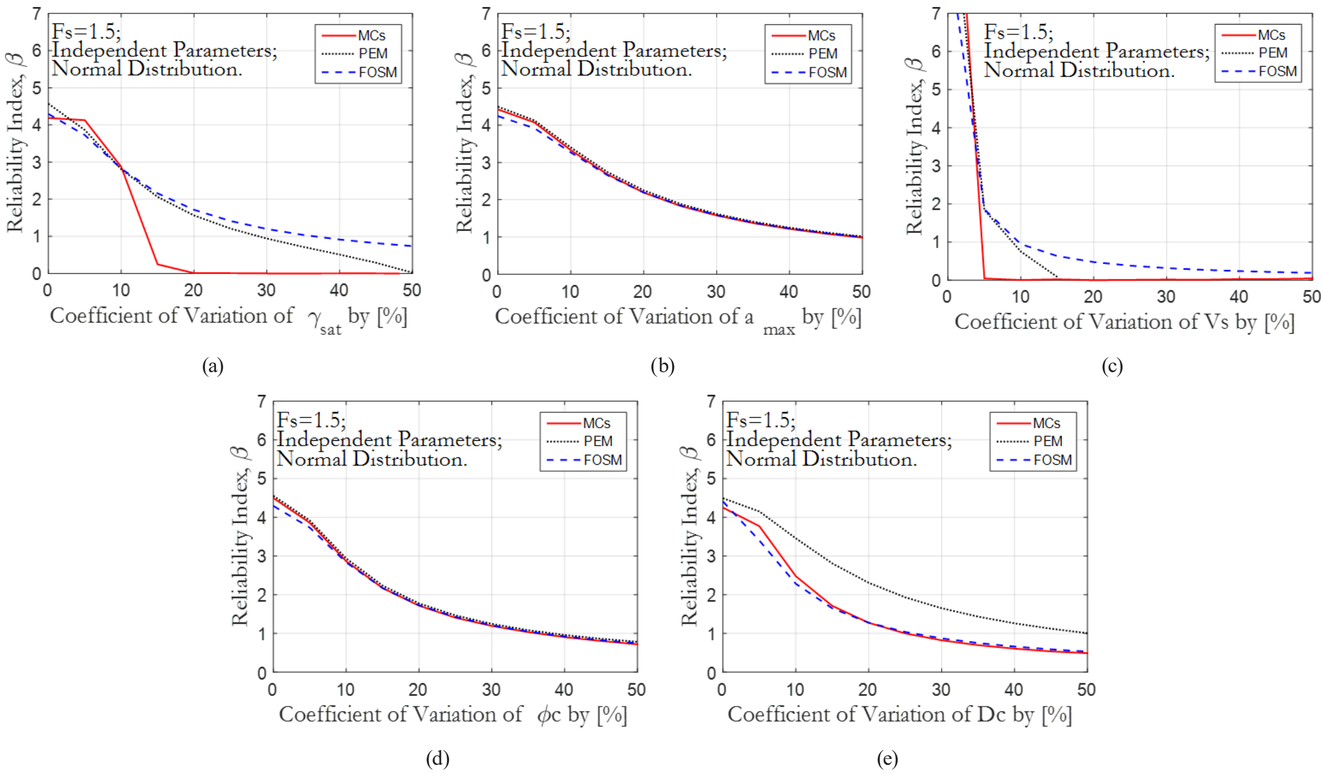


Fig. 7 Influence of the COV of individual parameters on the β for liquefaction analysis with normal distribution. The COVs of all other parameters were held at baseline values of 2% (very low uncertainty scenario): (a) Influence of the COV of γ_{sat} on β; (b) Influence of the COV of $a_{\max}$ on β; (c) Influence of the COV of V_s on β; (d) Influence of the COV of φ_c on β; (e) Influence of the COV of D_c on β

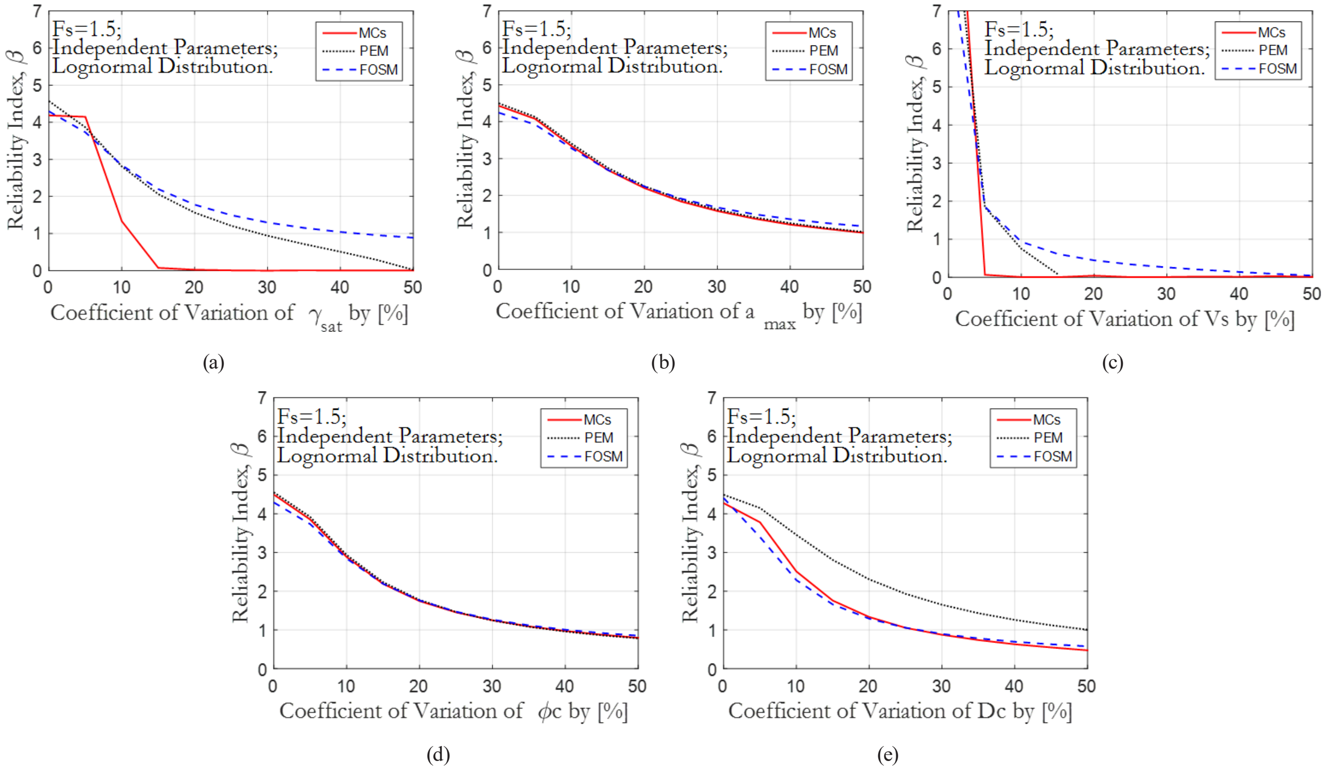


Fig. 8 Influence of the COV of individual parameters on the β for liquefaction analysis with lognormal distribution. The COVs of all other parameters were held at baseline values of 2% (very low uncertainty scenario): (a) Influence of the COV of γ_{sat} on β; (b) Influence of the COV of a_{max} on β; (c) Influence of the COV of V_s on β; (d) Influence of the COV of φ_c on β; (e) Influence of the COV of D_c on β

minimal differences between normal and lognormal distributions, indicating that the choice of distribution has negligible influence on β for the considered parameters.

In terms of probabilistic methods, results from PEM and FOSM closely match those of MCS for most variables. However, for high COV values of γ_{sat} and V_s, MCS exhibits noticeable errors due to redundancy issues linked to pseudo-random number generation and the non-linearity of the performance function. Additionally, for D_c, PEM slightly overestimates β compared to MCS and FOSM. Fig. 9 (a) to (f) illustrates the variation of β with COV for all parameters under different baseline uncertainty levels: 2% (very low), 5% (moderate), and 10% (high), using the PEM method for both normal and lognormal distributions.

The results confirm that increasing the overall uncertainty level from 2% to 10% amplifies the reduction in β across all parameters, particularly for V_s and γ_{sat}, where β can approach zero or even negative values under high uncertainty conditions, while a_{max}, φ'_c, and D_c remain relatively stable. Moreover, the comparison between normal and lognormal distributions shows negligible differences, indicating that the choice of distribution has little effect on β regardless of uncertainty level.

These findings highlight the critical impact of parameter uncertainties on the reliability of stone column design for liquefaction mitigation. In particular, the analysis demonstrates that even when the deterministic factor of safety suggests a safe design, high variability in key parameters such as V_s or γ_{sat} can significantly reduce reliability, leading to a higher p_f. This underlines the necessity of incorporating probabilistic approaches in design to capture the influence of uncertainties and ensure a more robust and risk-informed solution.

6 Sensitivity analysis and reduction of random variables

The study of the sensitivity of random variables of any limit state is very useful. This is justified by the fact that the change in the average value, as well as the coefficient of variation of a given variable can possibly induce significant modifications in the values of the reliability indices.

Based on the Kamien [45], the sensitivity analysis demonstrates the relative importance of random variables in influencing reliability. In this study, the omission sensitivity coefficient is applied, which indicates the relative error in the β when a random variable is replaced by its deterministic mean value. The coefficient of omission for a random variable is expressed by:

$$\Omega_i = \left|\beta - \beta_{X_i - x_i}\right|, \quad (21)$$

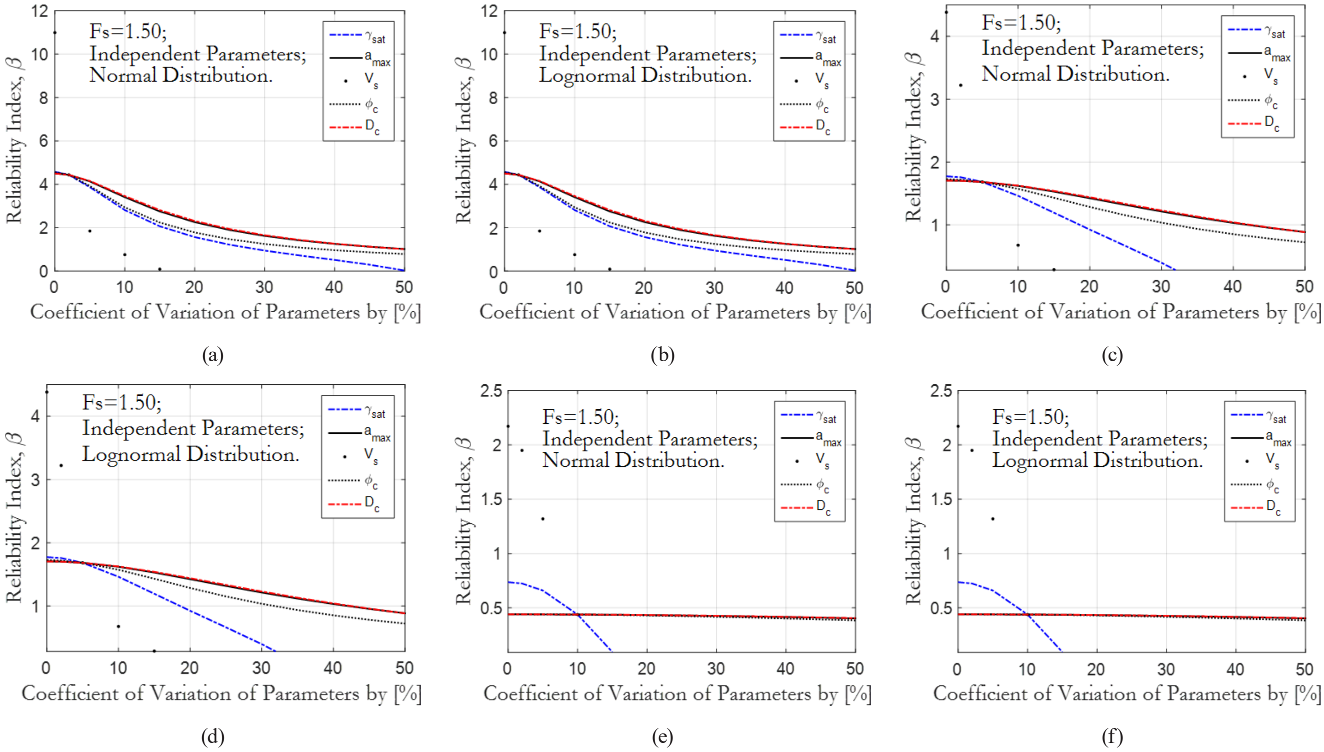


Fig. 9 Variation of the β versus the coefficient of variation (COV) of individual parameters in the stone column model for liquefaction mitigation. The COVs of all other parameters were held at baseline values of: (a) 2% (very low uncertainty, normal distribution); (b) 2% (very low uncertainty, lognormal distribution); (c) 5% (moderate uncertainty, normal distribution); (d) 5% (moderate uncertainty, lognormal distribution); (e) 10% (high uncertainty, normal distribution); (f) 10% (high uncertainty, lognormal distribution)

where β_i corresponds to the calculation of the β where all the variables are random, except $\boldsymbol{X}_i$ which is deterministic of a value x_i. β is the reliability index when all variables are probabilistic.

The aim of this study is to assess the sensitivity of the variables and to verify whether it is possible to identify those that have an insignificant influence on the reliability analysis.

Table 2 summarizes the parametric studies necessary for the sensitivity analysis. The mean friction angle was changed from Case 1 to Case 2, and the COV for all parameters were systematically varied as shown in Table 2.

Table 2 COV and mean values considered in the sensitivity analysis of liquefaction variables

Parameter	Symbol	Mean		COV (%)	
		Case 1	Case 2	Case 1	Case 2
Shear wave velocity	V_s	200 m/s	200 m/s	5	10
Stone friction angle	φ_c'	40°	38°	0–20	0–20
Saturated unit weight	γ_{sat}	21 kN/m³	21 kN/m³	10	15
Column diameter	D_c	1 m	1 m	10	20
Seismic acceleration	a_{max}	0.25g	0.25g	10	15
Percentage of fines	F_c	15%	15%	0	0

Section 6 focuses on reducing the number of random parameters required for the reliability analysis of stone column liquefaction by identifying the most and least significant parameters, thereby minimizing computational effort.

In order to assess the sensitivity of the random liquefaction variables, four series of calculations were carried out by fixing the mean and varying the COV of the φ_c', while keeping the COV of the other variables constant but different in each case. These calculations were verified by applying the laws of normal distribution. The full reliability analysis considered five principal random variables (a_{max}, γ_{sat}, D_c, φ_c', and V_s), while Fig. 10 presents results for two reduced combinations identified as most influential:

1. φ_c' and V_s;
2. φ_c', V_s, and γ_{sat}.

Reliability indices were computed for each combination, and omission factors were derived to evaluate the impact of excluding variables. The results show that the second combination (φ_c', V_s, and γ_{sat}) yields reliability indices that closely match those from the full five-variable set (reference combination), indicating that the excluded variables (D_c and a_{max})

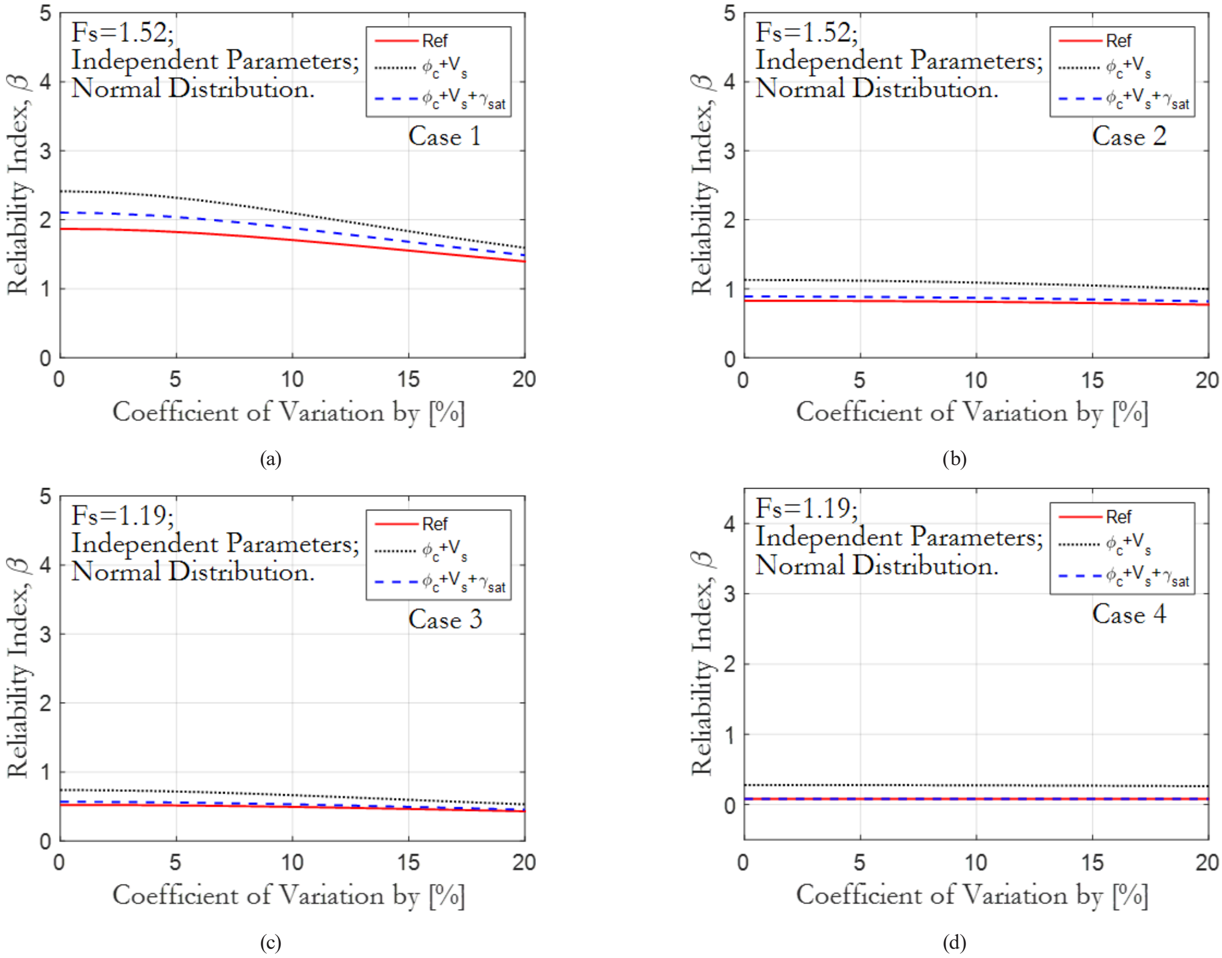


Fig. 10 Reduction random parameters in reliability analysis: (a) Series 1: φ_c' mean = 40°, varying COV of φ_c' with baseline COVs of other variables (Case 1); (b) Series 2: φ_c' mean = 40°, varying COV of φ_c' with different fixed COVs of other variables (Case 2); (c) Series 3: φ_c' mean = 38°, varying COV of φ_c' with baseline COVs of other variables (Case 3); (d) Series 4: φ_c' mean = 38°, varying COV of φ_c' with different fixed COVs of other variables (Case 4)

have minimal influence, as confirmed by omission factors below 0.5. This validates the acceptability of reducing the number of random variables, with φ_c', V_s, and γ_{sat} identified as the most critical parameters whose uncertainties must be carefully considered in reliability assessments.

Table 3 summarizes the results from the sensitivity study of the variables used in the probabilistic analysis for reducing liquefaction risk. From Table 3, it can be observed that an acceptable reduction in the number of random variables can only be achieved if the internal friction angle of the stone, V_s, and the γ_{sat} are treated as random variables. These parameters are considered essential for evaluating reliability, given that the omission factor is less than 0.5. In contrast, the D_c and $a_{\max}$ can be considered deterministic.

7 Validation and limitations of the study

The reliability-based analysis presented in this study was validated by adopting parameters and conditions from the real case study reported by Al-Homoud and Degen [9], which is widely cited as a reference for liquefaction mitigation using stone columns in silty sand deposits under seismic loading. The material properties, column geometry,

Table 3 Sensitivity analysis summary

Random parameters combination used	Reduction				Appropriate random parameters combination
	Case 1	Case 2	Case 3	Case 4	
$\varphi_c' + V_s$	▲	●	●	●	$\varphi_c' + V_s + \gamma_{sat}$
$\varphi_c' + V_s + \gamma_{sat}$	●	●	●	●	

●: Omission factor less than 0.5 (the use of this combination is without risk)
▲: Omission factor greater than 0.5 (the use of this combination represents a risk)

and loading conditions were selected to ensure consistency with this practical application. Furthermore, the computational framework and performance function formulation used in this study rely on well-established methods, including the Seed and Idriss [30] approach for liquefaction evaluation, ensuring that the results are aligned with recognized design principles.

However, certain limitations should be acknowledged. The analysis assumes uncorrelated random variables due to the absence of reliable correlation data and considers soil properties as normally or lognormally distributed under idealized homogeneous conditions. Additionally, the analysis considered a simplified seismic loading without incorporating time-history effects, which may affect pore pressure generation and stress distribution. Furthermore, the potential variability caused by stone column installation methods, such as differences in vibration intensity or replacement ratios, was not considered in the analysis, which could influence the actual in-situ performance of the columns.

Addressing these limitations in future research is essential to improve both the accuracy and practical relevance of reliability-based liquefaction assessments for improved ground.

8 Conclusions

This paper addresses the impact of uncertainties in the geotechnical design of stone columns through a reliability analysis for liquefaction mitigation using a probabilistic approach. In conclusion, based on the results and analyses presented, the following key points can be drawn from this study:

- For safe and economical designs of stone columns to mitigate liquefaction, a probabilistic approach is recommended to account for uncertainty effects.
- The reliability level decreases as the range of uncertainties increases; therefore, minimizing uncertainties as much as possible is necessary for a reliable design.
- The β is not significantly affected by whether the random variables follow a normal or lognormal distribution.
- The friction angle of the stone, the V_s, and the unit weight of saturated soil significantly influence the reliability analysis of stone columns designed to mitigate the risk of liquefaction.
- Based on the sensitivity analysis, certain parameters such as D_c, and $a_{\max}$ were treated as deterministic values, since their variation showed minimal impact on the β in the design. However, the internal friction angle of the stone, the V_s, and the unit weight of saturated soil were identified as essential random variables for the reliability assessment of the soil-column system related to liquefaction.
- Based on the results obtained, both FOSM and PEM reliability methods are practical tools for the reliability analysis of stone columns. The Monte Carlo method requires a large number of simulations and suffers from the redundancy problem, mainly when the COVs are high.

In addition to these findings, this study contributes to future research and practice by providing a unified reliability-based framework for evaluating the performance of stone column-improved ground. The approach demonstrates how probabilistic methods – such as MCS, FOSM, and PEM – can quantify the influence of parameter uncertainty on liquefaction mitigation effectiveness, enabling more rational and risk-informed design compared to traditional deterministic methods. For practitioners, the results highlight the critical role of parameters such as V_s, column geometry, and seismic loading in determining reliability, which can guide optimized design strategies and cost-effective ground improvement solutions. For future studies, the proposed framework can be extended to include soil heterogeneity, dynamic time-history analyses, and alternative improvement techniques, supporting the development of performance-based design for seismic ground improvement.

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