

# Research on the Torsional Mechanical Properties of Concrete Beams Reinforced with Carbon Fiber Reinforced Polymer and Angle Steel Combination

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## Abstract

Concrete beams are often subjected to torsion in the actual project, paste carbon fiber reinforced polymer and wrapped angle is commonly used in the actual project of the two concrete beam reinforcement methods, the use of carbon fiber reinforced polymer and angle for the combination of reinforcement, can be combined with the advantages of the two materials in the improvement of the structural mechanical properties, so that concrete beams can be reinforced to form a combination of superior performance of the structure, the concrete beams subjected to torsion beams for the combination of reinforcement is currently still At present, the combined reinforcement of concrete beams subjected to torsion is rarely involved. In this paper, for the carbon fiber reinforced polymer and angle steel combination of reinforced concrete torsion beams, designed 18 numerical specimens, its torsional stiffness and bearing capacity research, to determine the combination of torsional reinforced concrete beams of the main factors affecting the optimal combination of ratios, and to give the combination of reinforced concrete torsion beams torsional bearing capacity calculation formula. The research results show that the carbon fiber material and the splicing plate can form effective constraints on the concrete torsion beams, and the combined reinforcement method has a significant effect on the torsional load capacity, torsional stiffness and ductility enhancement of the reinforced concrete beams compared with the single reinforcement. The research results can provide technical support for the design and construction of combined reinforcement of concrete torsion columns.

## Keywords

concrete beams, carbon fiber reinforced polymer, angle steel, combination reinforcement, torsional bearing capacity

## 1 Introduction

Twisted components are a common type of basic load-bearing component, and the opening of holes in existing building renovation floor slabs can result in uneven load-bearing areas of beams, causing the beams at the edges of the openings to bear torque. The use of carbon fiber reinforced polymer to reinforce concrete torsion beams can restrain lateral deformation and improve the ductility of concrete [1–4], but its increase in torsional bearing capacity and stiffness of concrete beams is relatively small. The use of outsourced angle steel to reinforce concrete beams can significantly improve their torsional bearing capacity and stiffness [5–7], but has weak lateral deformation constraint ability. The use of carbon fiber reinforced polymer and angle steel combination to reinforce concrete torsion beams can leverage the advantages of both materials and

achieve better torsional reinforcement effect. At present, the combination of carbon fiber reinforced polymer and angle steel for strengthening reinforced concrete beams mainly focuses on bending and compression, and there is little research on torsional combination reinforcement. Wang et al. [8] and Yu and He [9] verified that the composite confinement concrete of carbon fiber reinforced polymer and angle steel enables the component to have good ductility and bearing capacity. Wang et al. [10] studied the composite reinforcement of reinforced concrete structures with carbon fiber reinforced polymer and angle steel, which can simultaneously leverage the advantages of both reinforcement materials. The stiffness degradation and load-bearing capacity degradation of the composite reinforced frame are relatively uniform, without sudden

changes, which can effectively prevent sudden collapse of the structure. Liu [11] verified that compared with carbon fiber reinforced polymer reinforcement and angle steel reinforcement, composite reinforcement has a greater delaying effect on the stiffness degradation of specimens. Lu et al. [12] and Zhao et al. [13] studied the eccentric compression and axial compression bearing capacity of concrete components after composite reinforcement, providing a theoretical basis for the application of this new composite reinforcement technology in engineering. There is still a lack of in-depth research on the collaborative working mechanism and torsional mechanical properties of reinforced concrete beams reinforced with carbon fiber reinforced polymer and angle steel combination.

Given the purpose of our study and the limitations of the experimental conditions, numerical studies provide us with a more flexible and controlled means to explore all aspects of the problem in depth. This article focuses on the finite element analysis and calculation of reinforced concrete torsion beams reinforced with carbon fiber reinforced polymer and angle steel combination as shown in Fig. 1, studies the effect of anti-torsion combination reinforcement, explores the working mechanism of anti-torsion combination reinforcement, determines the main factors affecting anti torsion combination reinforcement, and explores the calculation method of anti-torsion bearing capacity of reinforced concrete torsion beams reinforced with carbon fiber reinforced polymer and angle steel combination reinforcement.

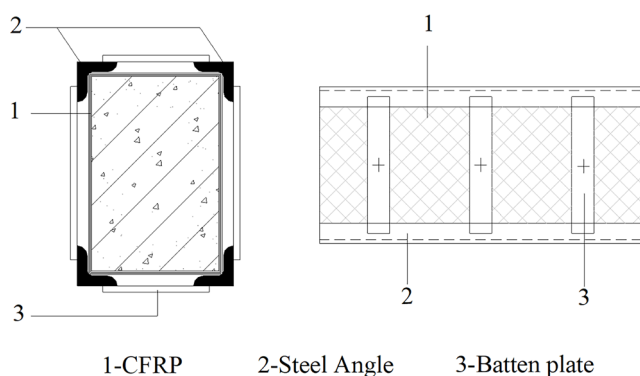


Fig. 1 Schematic diagram of combined reinforcement

## 2 Analysis of the torsional effect of composite reinforcement

### 2.1 Overview of the specimen

This article uses ANSYS software [14] to perform nonlinear analysis on the torsional performance of reinforced concrete rectangular section beams strengthened with carbon fiber reinforced polymer and angle steel combination, using a separated model. The size of the solid model is taken as  $b \times h \times l = 150 \times 250 \times 4000$  mm, the concrete strength grade is C30, SOLID65 three-dimensional solid elements are selected for the concrete, the failure criterion of the concrete adopts the William Warnke five parameter model, and the compressive stress-strain relationship of the concrete adopts the multi linear isotropic strengthening model [15].

The longitudinal reinforcement of the specimen is selected as HRB335, and the hoop reinforcement is selected as HPB300 grade steel bars. The diameter of the longitudinal reinforcement is 12 mm, and the cross-sectional area is 113.04 mm<sup>2</sup>. The diameter of the hoop reinforcement is 6 mm, and the cross-sectional area is 28.26 mm<sup>2</sup>. LINK180 three-dimensional chain element is selected for the steel reinforcement, and the stress-strain relationship of the steel reinforcement element is modeled using an ideal elastic-plastic model.

The carbon fiber adopts Shell41 elements with a design thickness of 0.111 mm. The angle steel model is  $\angle 30 \times \angle 30 \times 3$ , with a panel thickness of 4 mm and a spacing of 200 mm. Shell181 units are selected for angle steel and cladding, and the relevant material mechanical parameters are shown in Table 1.

The calculation model is a concrete beam under pure torsional stress state, with boundary conditions of consolidation at both ends, without considering its warping effect and the bond slip between various materials. The entire beam is subjected to uniformly distributed torque and is calculated using the incremental iterative method.

### 2.2 Authors and affiliations

To compare and analyze the bearing capacity of reinforced concrete torsion beams, this article analyzed three

Table 1 Selection of material mechanics parameters

Mechanical property	Concrete	Longitudinal reinforcement	Stirrup	Carbon fiber reinforced polymer	Steel angle/Batten plate
Elastic modulus ( $E$ /GPa)	13.585	200	200	235	200
Poisson's ratio ( $\nu$ )	0.2	0.3	0.3	-	0.3
Compressive strength ( $f_c$ /MPa)	14.3	-	-	-	-
Tensile strength ( $f_t$ /MPa)	1.43	335	300	5000	275.3

methods of strengthening: pasting carbon fiber reinforced polymer, wrapping angle steel, and combining carbon fiber reinforced polymer with angle steel, as shown in Fig. 2. The cracking torque and ultimate torque of each component were obtained, as shown in Table 2.

From Table 2, it can be seen that there is no significant change in the cracking torque of each component under the action of uniformly distributed torque. This is because in the pure torsional state, the main role of the cracking torque is determined by the torsional resistance of the concrete, and in the scope of this study, the role of carbon fiber reinforced polymer and angle steel is more reflected in improving the subsequent bearing capacity and ductility, rather than directly changing the cracking torque value. When the torque load reaches 5.1824 kN m, cracks begin to appear in the concrete. When the torque load reaches the ultimate torque, the program cannot continue iterating, that is, it reaches the torque value corresponding to the peak of the torque angle curve. Compared with single carbon fiber reinforced polymer reinforcement and angle steel reinforcement, the improvement is 56.25% and

237.5% respectively. The improvement of combined reinforcement reaches 562.5%, and the improvement effect of combined reinforcement is the most significant. Draw the torque and torsion angle variation curves during the analysis process of A1~A4 specimens, as shown in Fig. 3. From Fig. 3, it can be seen that the slope of the combined reinforcement is larger compared to other single reinforcement methods, indicating that the combined reinforcement has a better effect on improving the stiffness of concrete than the single reinforcement of carbon fiber reinforced polymer and angle steel. Composite reinforcement can allow for larger turning angles, indicating that when the short axis strain of concrete increases, carbon fiber reinforced polymer and angle steel participate in the torsion of the beam, significantly improving the torsional bearing capacity and ductility of the concrete beam.

### 3 Combined torsion resistance mechanism

There are many factors affecting the torsional mechanism of reinforced concrete torsion members reinforced with carbon fiber reinforced polymer and angle steel

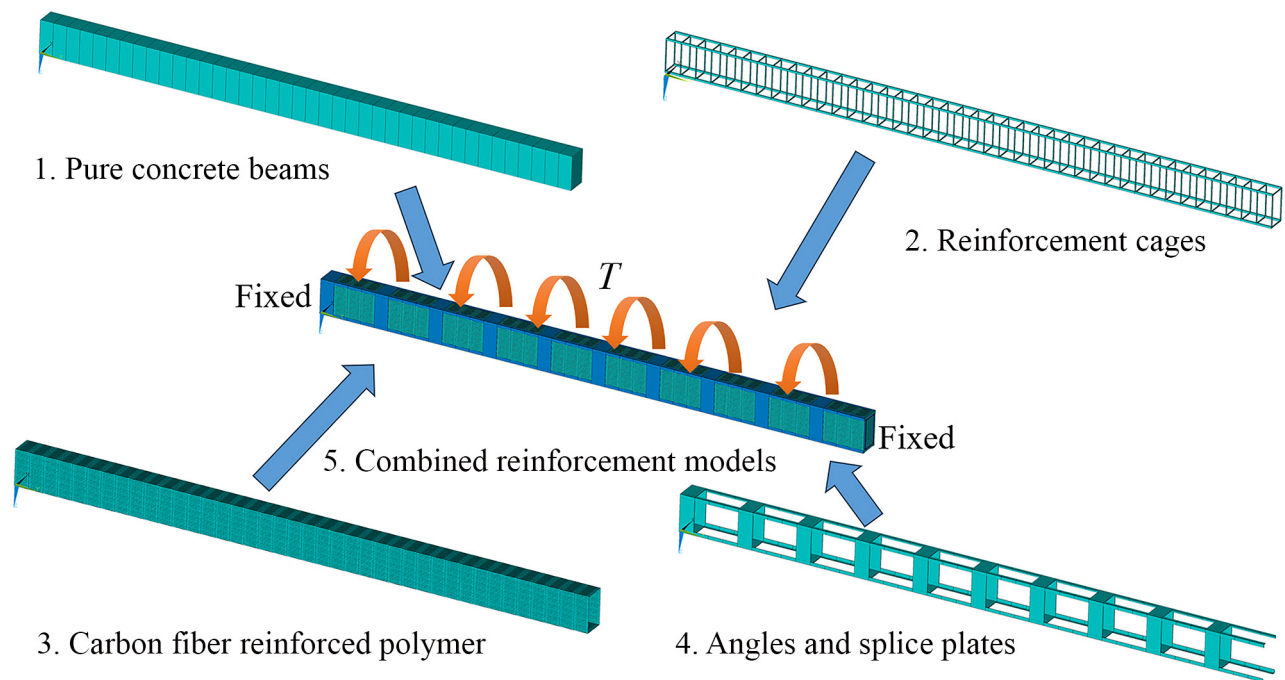


Fig. 2 Finite element model of combined reinforcement

Table 2 Data after loading finite element model

Specimen number	Reinforcement form	Cracking torque (kN m)	Ultimate torque (kN m)	Lifting range %
A1	Raw beam	5.1824	10.3648	-
A2	Carbon fiber sheet reinforcement	5.1824	16.195	56.25
A3	Angle reinforcement	5.1824	34.9812	237.5
A4	Combination reinforcement	5.1824	68.6668	562.5

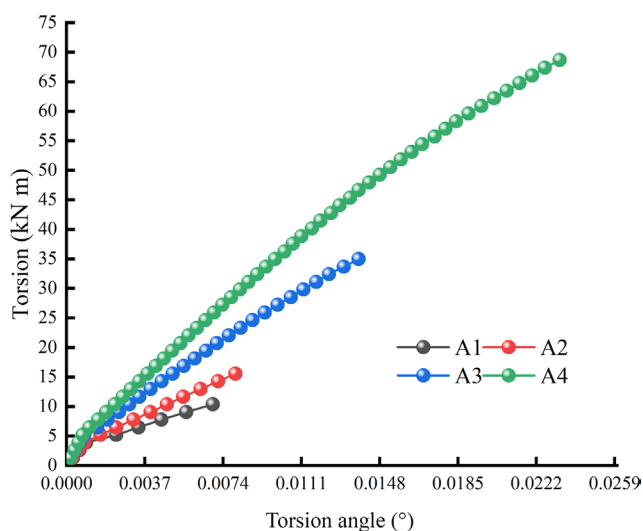


Fig. 3 Torque unit length torsion angle relationship curve for each specimen

combination, and the torsional mechanism is exceptionally complex [16]. For pure torsion members reinforced by combination, before cracking, the member is in an elastic working state, and the torque angle curve of the concrete beam basically overlaps and is linear, that is, the torque is mainly borne by the concrete part. At this stage, carbon fiber reinforced polymer and angle steel have almost no effect. As the load increases, when the component cracks and exhibits plasticity, the unit torsion angle of the component significantly increases. At the crack section, it exits work due to concrete cracking, and the stress and strain of the angle steel and carbon fiber reinforced polymer both significantly increase. For rectangular sections, the maximum principal tensile stress on the entire component section is located at the midpoint of the long side near both ends of the component. After cracking, the internal forces of the concrete beam are redistributed, the turning angle increases rapidly, and the stiffness of the concrete beam significantly decreases. Carbon fiber reinforced polymer and cladding begin to bear the load due to their strong suppression of the propagation of torsional cracks. As the load further increases, the steel bars and angle steel reach the yield strength and yield, and the deformation of the components further increases. The stress of the carbon fiber reinforced polymer also increases continuously. However, the concrete components reinforced with carbon fiber reinforced polymer still have a significant increase in torque bearing capacity, significantly improving the ductility and deformation performance of the twisted components. The carbon fiber reinforced polymer and angle steel of reinforced concrete torsion members with composite

reinforcement function similarly to the steel bars in torsion members, that is, the carbon fiber reinforced polymer and tie plate serve as hoop reinforcement, and the angle steel serves as longitudinal reinforcement. The combination reinforcement of reinforced concrete torsion members and the improvement of their torsional bearing capacity are mainly due to the effective restraint of carbon fiber reinforced polymer materials on concrete members. When carbon fiber reinforced polymer and angle steel work together, the restraint effect is the best and the material strength is fully utilized.

#### 4 Main influencing factors of composite reinforcement

In order to further investigate the effect of carbon fiber reinforced polymer and angle steel combination reinforcement on the torsional bearing capacity of concrete beams, 18 numerical specimens were designed with the number of carbon fiber reinforced polymer layers, angle steel types, and spacing between cladding plates as research variables. The specific parameters are shown in Table 3.

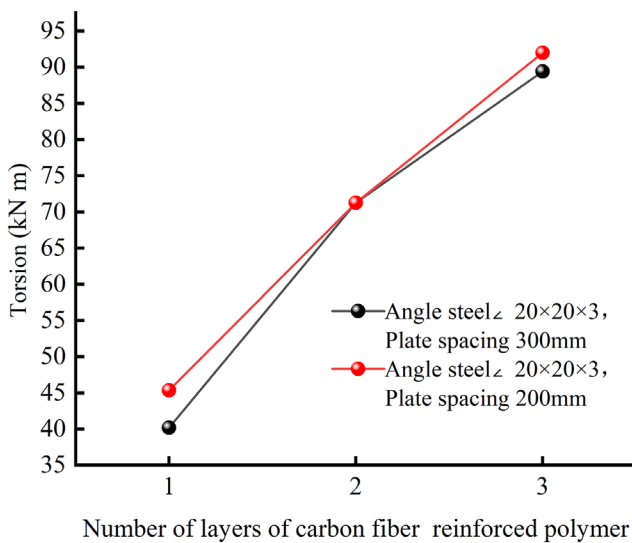
##### 4.1 Number of layers of carbon fiber reinforced polymer

Taking the number of layers of carbon fiber reinforced polymer as a variable and keeping other factors constant, the finite element calculation results in Fig. 4 show that when the number of layers of carbon fiber reinforced polymer varies between 1 and 3, the torsional ultimate bearing capacity of the reinforced concrete beam with carbon fiber reinforced polymer and angle steel combination increases linearly, and the slope decreases. This indicates that as the number of layers of carbon fiber reinforced polymer bonding increases, the torsional bearing capacity of the component also increases, but the two are not proportional. The more layers of bonding, the smaller the increase in single-layer material.

Take numerical specimen TB1-1-1 TB2-1-1, TB3-1-1, the torque and torsion angle curves were plotted as shown in Fig. 5. It can be seen from Fig. 5 that as the number of layers of carbon fiber reinforced polymer increases, the ultimate bearing capacity increases from 45.346 kN m at one layer to 91.9876 kN m at three layers, an increase of 102.86%, and the torsion angle increases from 0.01536° to 0.02552°, an increase of 53.12%. Therefore, carbon fiber reinforced polymer in composite reinforcement can improve the torsional bearing capacity of concrete beams and constrain lateral deformation to improve the ductility of concrete beams.

**Table 3** Loading data of finite element models with different combinations

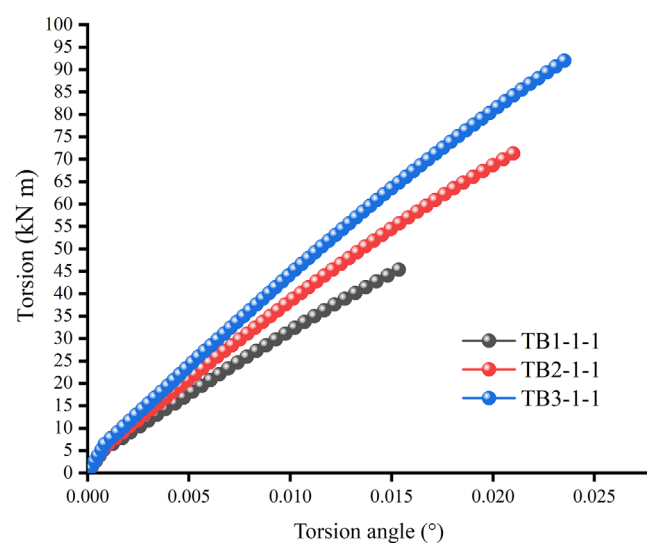
Specimen number	Number of layers of carbon fiber reinforced polymer	Angle type (mm)	Plate spacing (mm)	Ultimate torque (kN m)	Lifting range %
TB1-1-1	1	20 × 20 × 3	200	45.3460	12.90
TB1-1-2	1	20 × 20 × 3	300	40.1636	-
TB1-2-1	1	30 × 30 × 3	200	68.6668	70.97
TB1-2-2	1	30 × 30 × 3	300	51.8240	29.03
TB1-3-1	1	40 × 40 × 3	200	69.9624	74.19
TB1-3-2	1	40 × 40 × 3	300	51.8240	29.03
TB2-1-1	2	20 × 20 × 3	200	71.2580	77.42
TB2-1-2	2	20 × 20 × 3	300	71.2580	77.42
TB2-2-1	2	30 × 30 × 3	200	80.3272	100.00
TB2-2-2	2	30 × 30 × 3	300	76.4404	90.32
TB2-3-1	2	40 × 40 × 3	200	93.2832	132.20
TB2-3-2	2	40 × 40 × 3	300	66.0756	64.52
TB3-1-1	3	20 × 20 × 3	200	91.9876	129.00
TB3-1-2	3	20 × 20 × 3	300	89.3964	122.50
TB3-2-1	3	30 × 30 × 3	200	101.0568	151.60
TB3-2-2	3	30 × 30 × 3	300	94.5788	135.50
TB3-3-1	3	40 × 40 × 3	200	111.4216	177.40
TB3-3-2	3	40 × 40 × 3	300	106.2392	164.50



**Fig. 4** The effect of carbon fiber reinforced polymer on ultimate torque

#### 4.2 Angle steel model

When laying a layer of carbon fiber reinforced polymer as shown in Fig. 6, the selection of angle steel specifications has a small impact on the torsional bearing capacity, while carbon fiber reinforced polymer has a large impact on the unit torsional angle. As the number of layers of carbon fiber reinforced polymer increases, the influence of angle steel models on the ultimate bearing capacity gradually becomes apparent, and the slope increases. Therefore, in order to better demonstrate the role of angle steel in composite reinforcement, the article selects numerical



**Fig. 5** Torque unit length torsion angle relationship curve for different layers of carbon fiber reinforced polymer

specimens TB3-1-1, TB3-2-1, and TB3-3-1 to analyze their torque and torsion angle.

As shown in Fig. 7, the torsional bearing capacity increased from 91.9876 kN m to 111.4216 kN m, an increase of 21.13%, while the turning angle increased from 0.02352° to 0.02354°, an increase of only 0.08%. From this, it can be seen that the change in angle steel type has a significant impact on the bearing capacity, but has a weak ability to constrain lateral deformation.



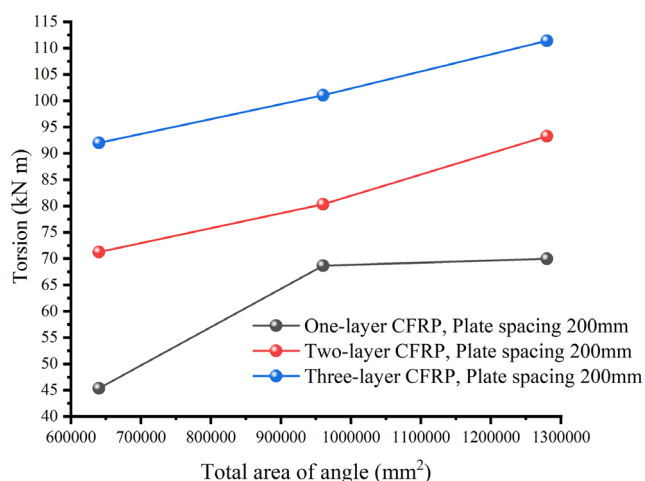


Fig. 6 The influence of the number of layers of carbon fiber reinforced polymer on the type of angle steel

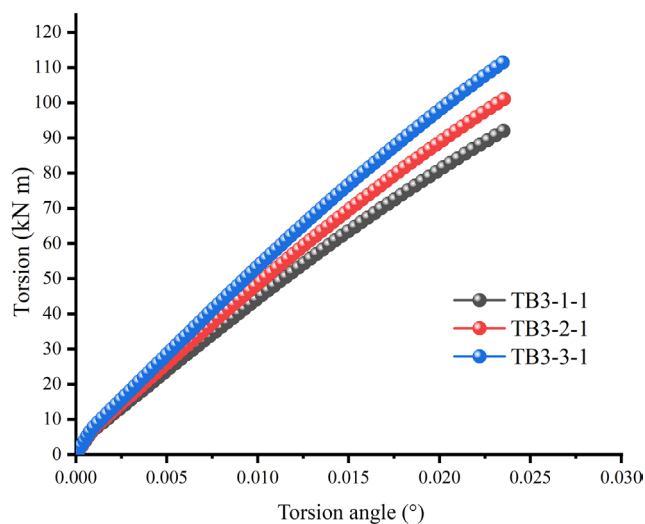


Fig. 7 Torque unit length torsion angle relationship curve for different types of angle steel

#### 4.3 Spacing between panels

Select all numerical specimens of one layer of carbon fiber reinforced polymer, unify the number of layers of carbon fiber reinforced polymer and the type of angle steel, and divide them into three groups of control specimens to analyze their torque and torsion angle as shown in Fig. 8.

Analysis found that the three sets of data have the same pattern. As the spacing between the panels increases, the bearing capacity decreases from 45.1636 kN m to 40.1636 kN m, 68.6668 kN m to 51.824 kN m, and 69.9624 kN m to 51.824 kN m, respectively. The torsional bearing capacity of the composite reinforced concrete decreases linearly and shows a negative correlation. As the amount of angle steel increased by, for the two groups of specimens TB1-2-1 and TB1-2-2, as well as TB1-3-1 and TB1-3-2, the spacing between the plates decreased and the

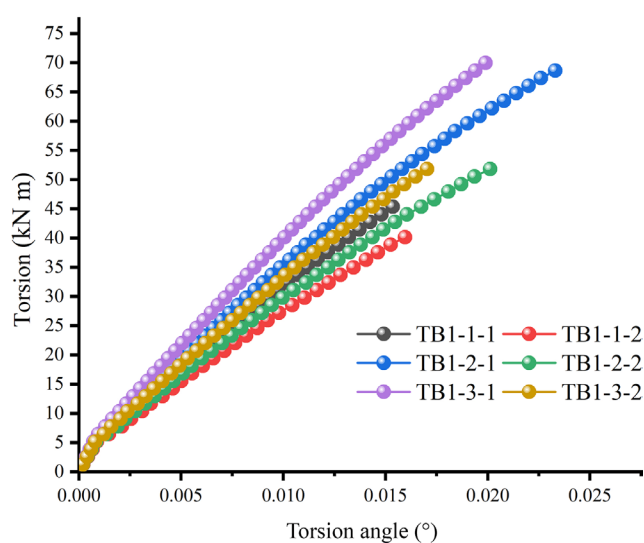


Fig. 8 The effect of panel spacing on ultimate torque

torsion angle increased significantly, with the first group increasing by 15.86% and the second group increasing by 16.79%. Therefore, in torsional reinforcement, the panel exhibits the same mechanism of action as carbon fiber reinforced polymer, which can effectively restrain lateral deformation and improve the ductility of concrete.

Furthermore, from Table 3 and finite element analysis, it can be concluded that when the number of layers of carbon fiber reinforced polymer is one in torsional reinforcement, the influence of angle steel type on the torsional bearing capacity is relatively small, and the spacing between panels is the main influencing factor. As the number of layers of carbon fiber reinforced polymer increases, the influence of angle steel type on the ultimate bearing capacity gradually becomes apparent. In the study of the three variables, it was found that the type of angle steel and the number of layers of carbon fiber reinforced polymer have a significant impact on the ultimate bearing capacity, while the number of panels and layers of carbon fiber reinforced polymer have a significant improvement on ductility.

#### 5 Calculation of torsional bearing capacity of reinforced concrete beams with composite reinforcement

In the spatial truss model, concrete torsion members form a torsion spatial truss after cracking under torsion. Assuming that the concrete between the cracks no longer bears tensile stress and can only bear pressure, it is considered as a compressed diagonal web member of the truss. The tensile stress is then borne by steel bars, carbon fiber reinforced polymer, and angle steel, which form the tension web members and chord members of the spatial truss.

The oblique pressure field formed by the pressure inside the concrete diagonal compression rod bears the torque. According to the theory of thin-walled torsion, it is known that the internal pressure of a diagonal brace is not uniformly distributed from the inside to the outside of the wall. At the same time, it is assumed that the carbon fiber reinforced polymer, angle steel, and the surface concrete of the component form a whole, coordinating deformation without considering the relative slip at the interface between the carbon fiber reinforced polymer, angle steel, and concrete [17–19].

Carbon fiber reinforced polymer and angle steel are used to strengthen the torsional reinforced concrete members. The carbon fiber reinforced polymer, plate and angle steel are all subjected to tensile stress, that is, the tension rod of the space truss is strengthened.

The reinforcement term of the formula for calculating the torsional bearing capacity of reinforced concrete members under pure torsional action in China's GB 50010-2010 Code for Design of Concrete Structures [20] can be modified. The carbon fiber reinforced polymer used in this paper is a fully wrapped form, and the change of its layer number is mainly studied. The parameter  $\alpha$  is introduced to represent the reduction coefficient of the layer number of carbon fiber reinforced polymer. The torsional bearing capacity formula of the revised GB 50010-2010 Code for Design of Concrete Structures [20] is changed into the following form:

$$T_u = 0.35 f_t W_t + 1.2 \sqrt{\xi'} \times \left[ \frac{f_{yv} A_{svl} A_{cor}}{s_v} + \alpha \frac{f_{cf,v} (n+1) A_{cf,vl} A}{s_v} + \frac{f_{cf,t} A'_{cf,vl} A}{s_{cf,t}} \right] \quad (1)$$

$$\xi' = \frac{\frac{A_{st} f_y}{u_{cor}} + n \frac{A_{cf,l} f_{cf,l}}{u} + \frac{A'_{cf,l} f_{jg,l}}{u}}{\frac{A_{svl} f_{yv}}{s_v} + n \frac{A_{cf,vl} f_{cf,v}}{s_v} + \frac{A'_{cf,vl} f_{cf,t}}{s_{cf,t}}} \quad (2)$$

In the formula:

- $T_u$  – design value of torsional capacity of CFRP reinforced member;
- $f_t$  – design value of tensile strength of concrete;
- $W_t$  – shear surface torsion resistance moment, rectangular cross-section calculated as;  
 $W_t = \frac{b^2}{6} (3h - b)$
- $\xi'$  – strength ratio between longitudinal and transverse fiber reinforced steel and steel;

- $f_y, f_{yv}$  – design value of tensile strength of longitudinal bar and stirrup;
- $f_{cf,l}, f_{cf,v}$  – effective tensile stress of longitudinal and transverse carbon fiber reinforced polymer strips;
- $f_{jg,l}, f_{cf,t}$  – design value of tensile strength of angle steel and plate;
- $u, u_{cor}$  – the perimeter of the section and the perimeter of the section core;
- $A, A_{cor}$  – the gross area of the section and the area of the core of the section;
- $A_{st}, A_{svl}$  – the gross area of the longitudinal reinforcement and the area of the stirrup of each limb;
- $A_{cf,l}, A_{cf,vl}$  – the total area of longitudinal single carbon fiber strip and the area of each single horizontal carbon fiber strip;
- $A'_{cf,l}, A'_{cf,vl}$  – the total area of the angle steel and the area of each plate;
- $s_v, s_{cf,t}$  – stirrup spacing and plate spacing;
- $n$  – number of layers of carbon fiber reinforced polymer;
- $\alpha$  – the reduction coefficient of the number of layers of carbon fiber reinforced polymer is 1.2 for one layer, 0.95 for two layers, and 0.85 for three layers.

It can be seen from Eq. (1) that the torsional ultimate bearing capacity is mainly related to the amount of steel with fiber reinforcement and the properties of the material itself. The torsional ultimate bearing capacity increases when the ratio of carbon fiber sheet to angle steel increases. The torsional bearing capacity decreases with the increase of the spacing of the plates.

## 6 Computational model verification

A hole needs to be opened in the reconstruction of an existing building, so that the side beam is in a state of torsion. The section of the beam  $b \times h = 200 \times 300$  mm, the span  $l = 4000$  mm, the design strength of the concrete is C30, the longitudinal reinforcement is 4Ø12, the stirrup is Ø8@100, and the original torque of the beam is calculated to be 12 kN m. After opening the hole, the torque shall be 70 kN m. The beam is reinforced with carbon fiber reinforced polymer and angle steel. The carbon fiber reinforced polymer is fully wrapped and laid in a single layer with a thickness of 0.111 mm. The angle steel type is  $\angle 20 \times 20 \times 3$ , the thickness of the plate is 3 mm, and the spacing is 200 mm. The relevant material mechanics parameters are shown in Table 4.

**Table 4** Mechanical parameters of materials

Mechanical property	Concrete	Longitudinal reinforcement	Stirrup	Carbon fiber reinforced polymer	Steel angle
Elastic modulus ( $E/\text{GPa}$ )	13.585	200	200	235	200
Poisson's ratio ( $\nu$ )	0.2	0.3	0.3	-	0.3
Tensile strength ( $f_t/\text{MPa}$ )	1.43	455	300	3510	375.4

Now, the above parameters are brought into Eq. (1) to obtain the strength ratio coefficient of longitudinal fiber reinforced steel and transverse fiber reinforced steel and the torsion plastic resistance moment of the section.

$$\xi' = \frac{\frac{452.16 \times 455}{0.84} + 1 \times \frac{111 \times 3510}{1} + \frac{480 \times 375.4}{1}}{\frac{50.24 \times 300}{0.1} + 1 \times \frac{11.1 \times 3510}{0.1} + \frac{300 \times 375.4}{0.2}} = \frac{244920 + 389610 + 180192}{150720 + 389610 + 563100} = \frac{814722}{1103430} = 0.738$$

$$W_t = \frac{b^2}{6} (3h - b) = 4.67 \times 10^{-3}$$

By substituting the above parameters into Eq. (2), we can obtain:

$$T_u = 0.35 \times 1.43 \times 10^6 \times 4.67 \times 10^{-3} + 1.2 \sqrt{0.738} \times \left( \frac{300 \times 50.24 \times 0.0416}{0.1} + 1.2 \times \frac{3510 \times 2 \times 11.1 \times 0.06}{0.1} + \frac{375.4 \times 300 \times 0.06}{0.2} \right) = 101.457 \text{ kN m.}$$

In order to verify the accuracy and reliability of the combined reinforcement, finite element calculation is carried out. The limit value of the torsional bearing capacity after the combined reinforcement is 103.35 kN m, which is 1.87% higher than the theoretical calculation value in this paper. Compared with the torque 70 kN m required after reinforcement, the beam is increased by 47.64% and is in a safe state after reinforcement.

The above analysis shows that the formula given in this paper can accurately consider the torsional bearing capacity of reinforced concrete beams strengthened by the combination of different carbon fiber reinforced polymer ratio and angle steel dosage, and has high accuracy and reliability.

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## 7 Conclusions

The conclusions of this paper are:

1. The torsional load capacity of reinforced concrete beams and the ductility of beams are improved significantly by the combination of reinforcement methods compared with single reinforcement.
2. Based on the truss model, the torsional load capacity calculation formula of reinforced concrete beams reinforced with carbon fiber reinforced polymer and angle is given, and it is concluded that the torsional ultimate load capacity is mainly related to the amount of fiber and steel reinforcement and the nature of the material itself.
3. Splice plate and carbon fiber reinforced polymer in torsional reinforcement present the same mechanism, are able to better restrain lateral deformation to improve the ductility of concrete. The change of angle type has obvious effect on the bearing capacity, but it is weak in restraining the transverse deformation.
4. In the actual project, when the number of layers of carbon fiber reinforced polymer is one layer, it is recommended to choose a moderate angle steel type and small spacing of the plate. Because the number of layers of carbon fiber reinforced polymer is not proportional to the bearing capacity, the number of adhesive layers is large, and the material utilization rate is small, if it is necessary to greatly improve the torsional bearing capacity and ductility, it is recommended to use the combination of two layers of carbon fiber reinforced polymer and angle steel for reinforcement.

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