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DETAILED GEOID DETERMINATION USING THE COMBINATION OF TRUNCATED GLOBAL INTEGRALS AND GEOPOTENTIAL MODELS

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Abstract

In modern geodetic environment the GPS technique is a basic tool for high accuracy positioning. The GPS can be used also for height determination, but because of the different reference systems, this application requires geoid solutions as accurate as the GPS itself, that is better than 1 ppm. The investigation to realize such accurate geoid solution was started at the Satellite Geodetic Observatory in 1986 (ÁDÁM et al, 1988). Following the given gravity data distribution in this area, a special data truncation method was chosen which incorporates local terrestrial gravity data integration and global geopotential model contribution. The review of the data available and the method applied are described here together with the comparative analysis of the different solutions.

Keywords: geoid, gravimetry, GPS, deflections of the vertical.

Introduction

The high resolution and so-called 'cm-accuracy' geoid is an indispensable tool for the modern geodesy. The quickly spreading applications of the NAVSTAR Global Positioning System require geoid solutions accuracy comparable with GPS (SCHWARZ et al, 1985; ENGELIS et al, 1985) and the scientific role of the detailed geoid solutions would also be very important (e.g. geodynamics, geophysical interpretation). The research project started at the FÖMI Satellite Geodetic Observatory in co-operation with the Loránd Eötvös Geophysical Institute and partly with the financial support of the HAS OTKA Research Fund set the aim of realizing geoid solutions which are suitable for the modern geodetic purposes and requirements. Because of the special gravity data availability (dense data distribution in Hungary but scanty data over the border), the different truncation techniques (MOLODENSKY et al, 1962; WONG and GORE, 1969; OSTACH, 1970; MEISSL, 1971; WENZEL, 1982; JEKELI, 1982; SJÖBERG, 1984; HECK and GRÜNINGER, 1987; etc) were investigated. Among the numerous methods, the Meissl–Ostach type solution was chosen because of its simplicity and efficiency. The essence of the different truncation techniques is that the evaluation of the global integrals as the Stokes and Vening Meinesz integrals is confined to the truncation cap, and the effect of the remaining, so-called distant zone is taken into account in terms of the geopotential model coefficients. By using different truncation cap radii, five different solutions were completed in 1991.

Data

The different data sources, especially the gravity data from the neighbouring countries have different reference systems. During the pre-processing, all of the data were transformed into the common geocentric GRS80 reference ellipsoid and the IGSN71 Gravity System.

The Hungarian Gravity Survey (HGS)

The HGS consists of about 120.000 point gravity measurements and is owned by the Loránd Eötvös Geophysical Institute (ELGI). This Institute provided the gravity data for the computations in the form of interpolated (GRID.A) free-air gravity anomalies on a grid with 800 m mesh size (FRANKE, 1978; SZABÓ et al, 1989). The terrain correction term was neglected because presently the detailed terrain model is not available for geoid determination purposes. On the other hand, the maximal terrain effect on the geoid is estimate about 2 cm in Hungary.

Inner Zone Gravity Measurements

The computation of the gravimetric deflections of the vertical require the detailed knowledge of the gravity field in the close surroundings of the given points. During the last two decades, inner zone gravity measurements were carried out at more than 140 astronomical points.

Gravity Data from the Neighbouring Areas

The quality and distribution of these data are very uneven (see *Fig. 1*). The overborder gravity data were interpolated onto two grids: GRID.B: 4800 m mesh size – the rectangle covers the territory of Hungary.

The interpolation was done to grid knots within this rectangle but only over the Hungarian border.

GRID.C: 9600 m mesh size – the size of this rectangle allows the computations with truncation cap up to 4 degrees. *Fig. 2* shows the gravity field within the rectangle of GRID.B.

Spherical Harmonic Expansions of the Geopotential

Detailed comparisons were carried out between the different geopotential model solutions (ÁDÁM, 1991). According to these results, the OSU89B solution ($n, m = 360$) (RAPP and PAVLIS, 1990) proved to be the best for the territory of Hungary.

Astronomical Measurements

In the last 40 years, astronomical measurements at more than 120 points were carried out (LÉVAI, 1988). These measurements together with the computed gravimetric deflections are used for the combined astrogravimetric quasi-geoid solution according to the modified Molodensky's astrogravimetric levelling (BÖLCSVÖLGYI et al., 1987).

Satellite Doppler and GPS Measurements

The temporary height datum of the quasi-geoid solutions was served by the long term NNSS Doppler observations carried out at the SGO (ÁDÁM, 1987). The astronomical measurements were transformed into the geocentric GRS80 reference ellipsoid by Doppler observation (ÁDÁM, 1987) with the use of the Vening Meinesz transformation procedure. After having the Reference Point at the SGO new co-ordinates in the European Reference Frame (EUREF EAST'91 GPS Campaign), the GPS will replace these functions. The GPS measurements are presently used only for checking the relative accuracy of the different geoid solutions.

Method

In performing gravimetric geoid determinations, a real problem could be encountered: how to get reasonable gravity data over as a big part of the globe as possible. Unfortunately, this requirement cannot be fulfilled because of the natural (e.g. the lack of the data from the oceans) or financial or political endowments. The application of truncation allows using only gravity data within a relatively small truncation cap around

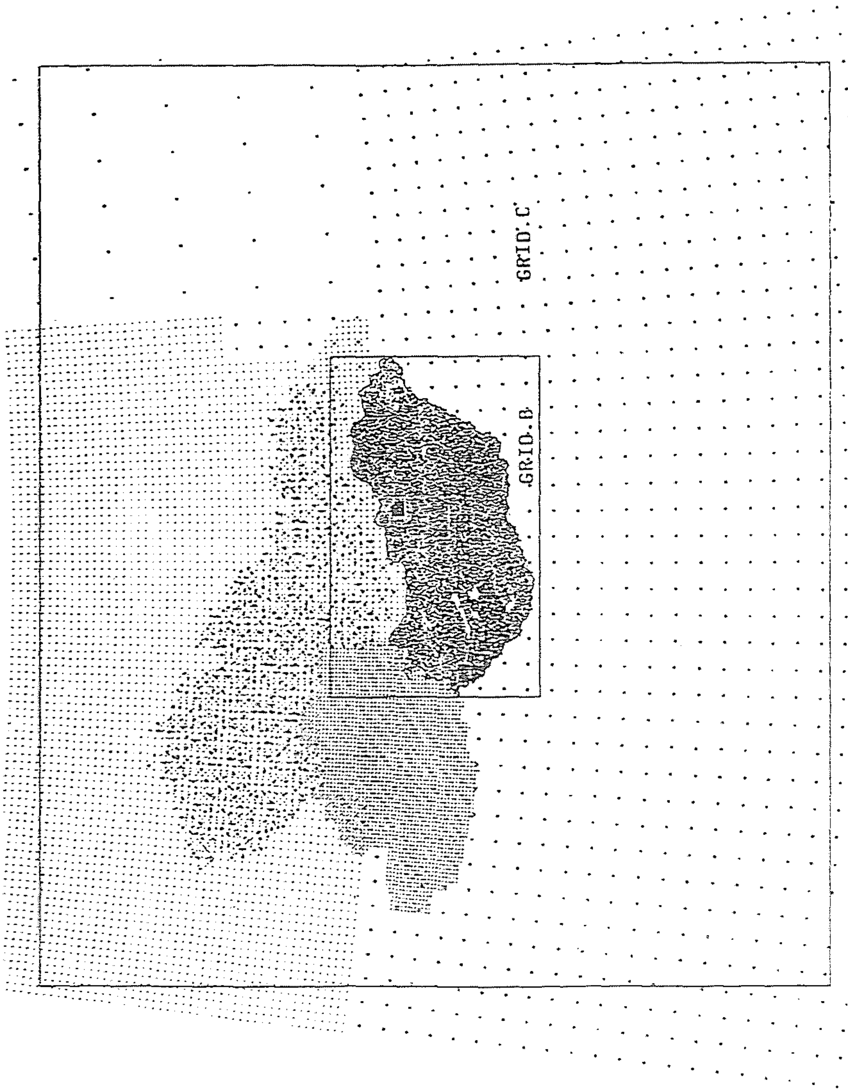

Fig. 1. The distribution of the gravity data available

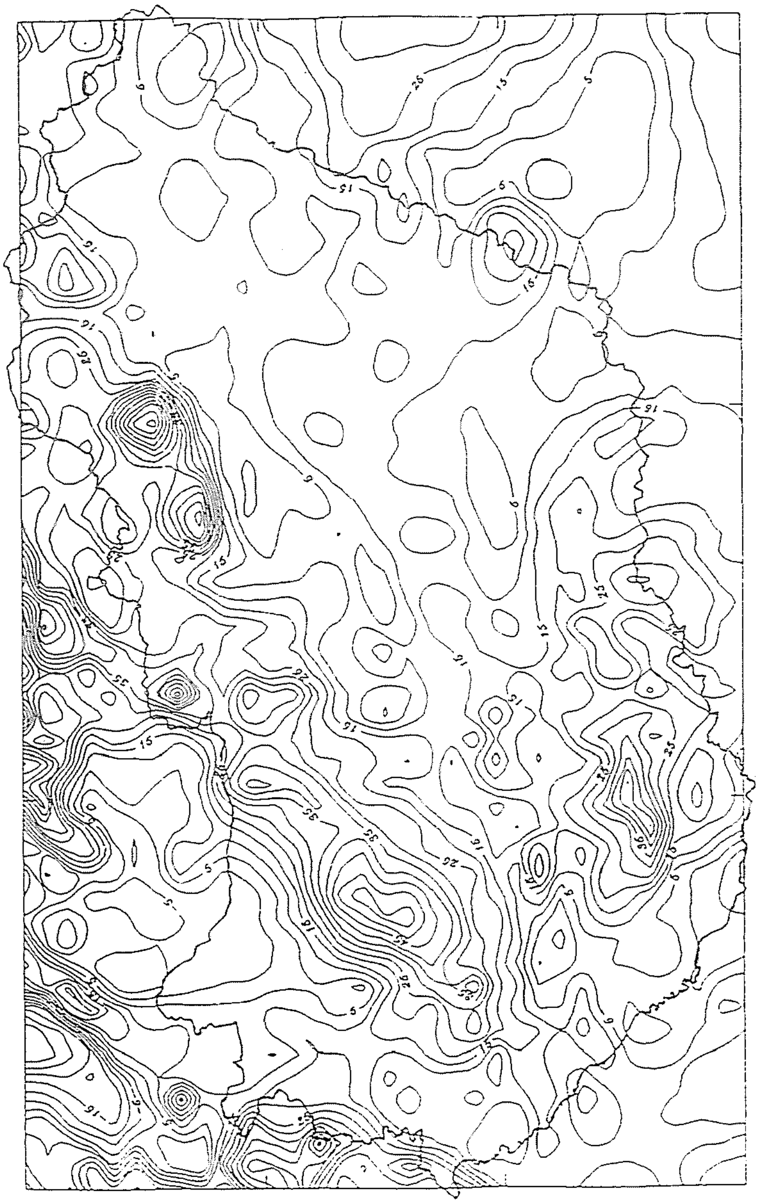

Fig. 2. The free-air anomaly map of Hungary. Contour interval 5 mgals

the computational point, and the effect of the remaining areas could be taken into account by using a spherical harmonic expansion model of the geopotential.

During the last three decades, numerous methods were developed based on different philosophies (MOLODENSKY et al, 1962; WONG and GORE, 1969; OSTACH, 1970; MEISSL, 1971; WENZEL, 1982; JEKELI, 1982; SJÖBERG, 1984; HECK and GRÜNINGER, 1987; GUAN and LI, 1990, etc). Among the solutions, the mathematically simplest one is the Meissl-Ostach type modification but its efficiency proved to be one of the best (WICHIEN-CHAROEN, 1984; ZHANG, 1990). The philosophy is really simple. The pure truncation causes a jump in the kernel functions $S(\psi)$ and $V(\psi)$ at the edge of the truncation cap ψ_0, which has serious negative effect on the frequency domain. In order to eliminate the aliasing effect, the value of the kernel function at the edge is subtracted from the kernel:

$$S^M(\psi) = S(\psi) - S(\psi_0),$$
$$V^M(\psi) = V(\psi) - V(\psi_0).$$

The Stokes and Vening Meinesz formulas according to the Molodensky's theory:

$$\zeta = \frac{R}{4\pi\gamma} \int_\sigma S(\psi) \Delta g' d\sigma, \tag{1}$$

$$\begin{bmatrix} \xi^G \\ \eta^G \end{bmatrix} = \frac{1}{4\pi\gamma} \int_\sigma V(\psi) \Delta g' \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} d\sigma, \tag{2}$$

where ζ : gravimetric height anomaly
R : mean earth radius
γ : mean earth gravity
$S(\psi)$: Stokes function
$\Delta g'$: terrain corrected free-air anomaly
ξ^G, η^G : gravimetric deflections of the vertical
$V(\psi)$: Vening Meinesz function

The Meissl–Ostach type modification:

$$\hat{\zeta} = \frac{R}{4\pi\gamma} \int_{\psi_0} S^M(\psi) \Delta g' d\sigma + \frac{R}{2\gamma} \sum_{n=2}^{N\max} \bar{Q}_n^H \Delta g_n, \tag{3}$$

$$\begin{bmatrix} \hat{\xi^G} \\ \hat{\eta}^G \end{bmatrix} = \frac{1}{4\pi\gamma} \int_{\psi_0} V^M(\psi) \Delta g' \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} d\sigma + \frac{1}{2\gamma} \sum_{n=2}^{N\max} \bar{Q}_n^D \begin{bmatrix} \frac{\partial \Delta g_n}{\partial \phi} \\ \frac{\partial \Delta g_n}{\cos\phi \partial \lambda} \end{bmatrix}, \tag{4}$$

where Δg_n: computed from the geopotential model
$\bar{Q}_n^H$: Meissl's truncation coefficients for height anomalies
$\bar{Q}_n^D$: Meissl's truncation coefficients for gravimetric deflection of the vertical
$N_{\max}$: maximal degree of the geopotential model

The truncation and the incorporation of a geopotential model cause an obvious frequency domain separation in the equations. The integral terms contain the short and medium wavelength part (*Fig. 3*), the summation terms contain the long wavelength part (*Fig. 4*) of the solution. The summation should be continued up to the infinity, but the spherical harmonic models of the geopotential incorporate terms only to $N_{\max}$. The high degree geopotential models (e.g. OSU solutions) usually contain terms up to degree 180 or 360. The effect of the lacking terms over $N_{\max}$ is called omission error and could be estimated by the global covariance model of the geopotential.

Results

The mathematical establishment was followed by an intensive computer program developmental phase, in order to realize the desired detailed gravimetric quasi-geoid solution for Hungary. The program system named as GEOMETR (GEOid determination based on the MEissl TRuncation method) was written in FORTRAN and can be run on IBM PC/AT 286–486 computers. The program system is capable of computing gravimetric geoidal height anomalies, gravimetric deflections of the vertical and astrogravimetric geoidal height anomalies. The simplified flowchart of the program system can be found in *Fig. 5*. Five different solutions were completed in 1991 (KENYERES, 1991), named as HGQ91A,B,C,D,E with the following parameters:

Table 1
Parameters of the HGQ91 geoid solutions

Solution	Truncation cap	Geop. degree
HGQ91A	$\psi = 0.25°$	$n, m = 360$
HGQ91B	$\psi = 0.50°$	$n, m = 360$
HGQ91C	$\psi = 1.00°$	$n, m = 360$
HGQ91D	$\psi = 2.00°$	$n, m = 360$
HGQ91E	$\psi = 4.00°$	$n, m = 360$

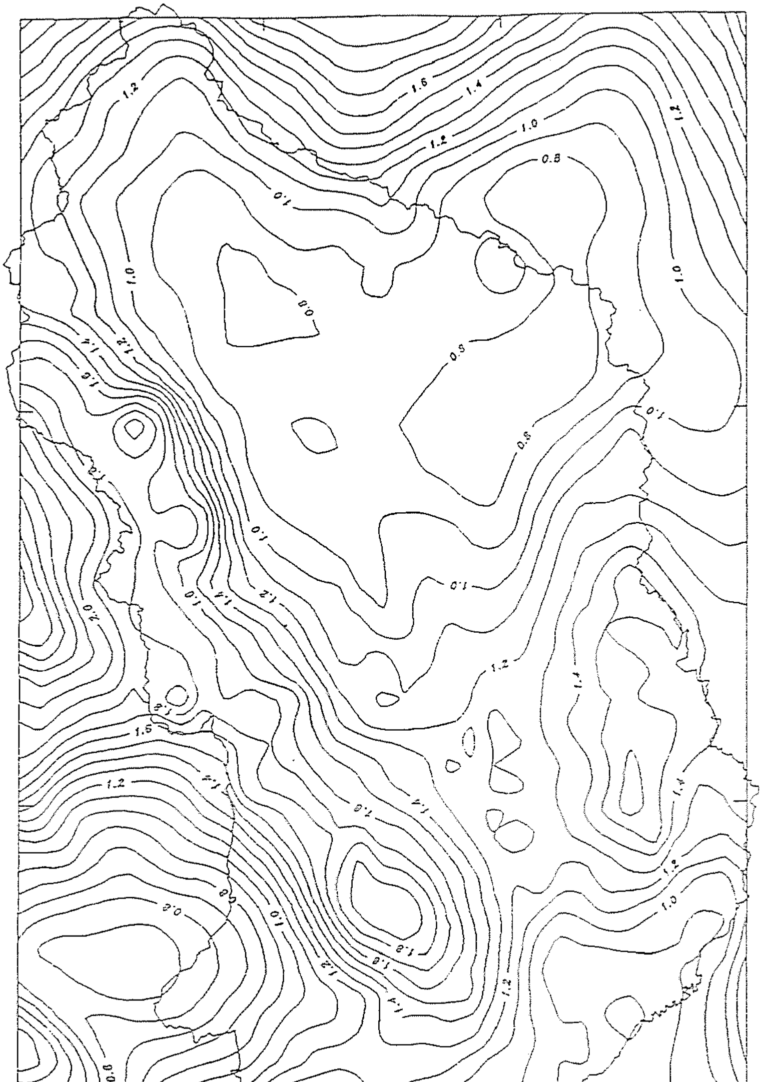

Fig. 3. The local gravity field contribution ($\psi_0 = 1°$) to the HGQ91C solution. Contour interval 0.1 m

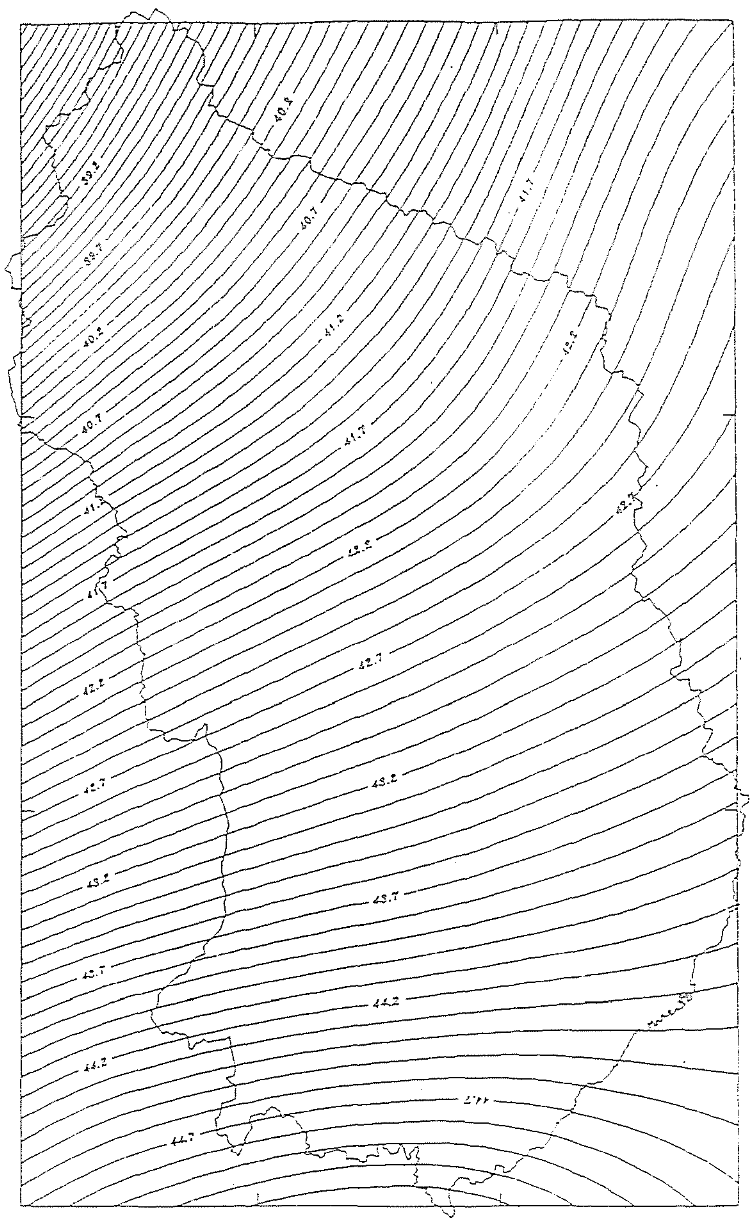

Fig. 4. The geopotential model (OSU89B $n, m = 360$) contribution to the HGQ91C solution. Contour interval 0.1 m

PROGRAM SYSTEM

GEOMETR: GEOid determination based on the MEissl TRuncation method

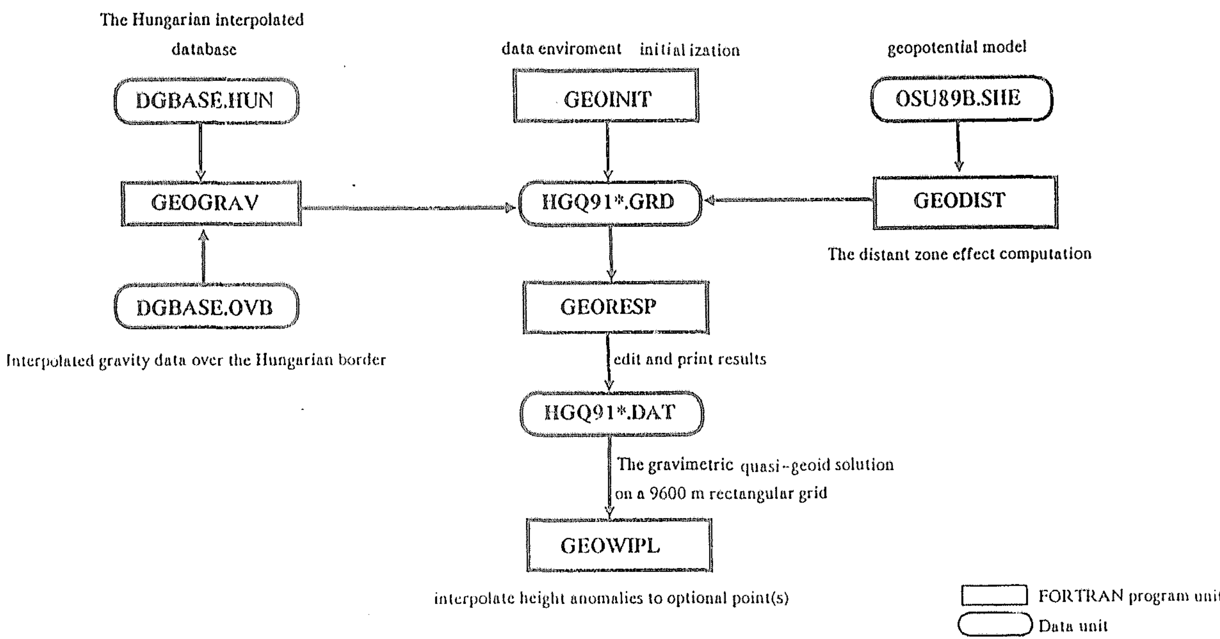

Fig. 5. Simplified flowchart of the GEOMETR program system without parts for astro-gravimetric geoid determination.

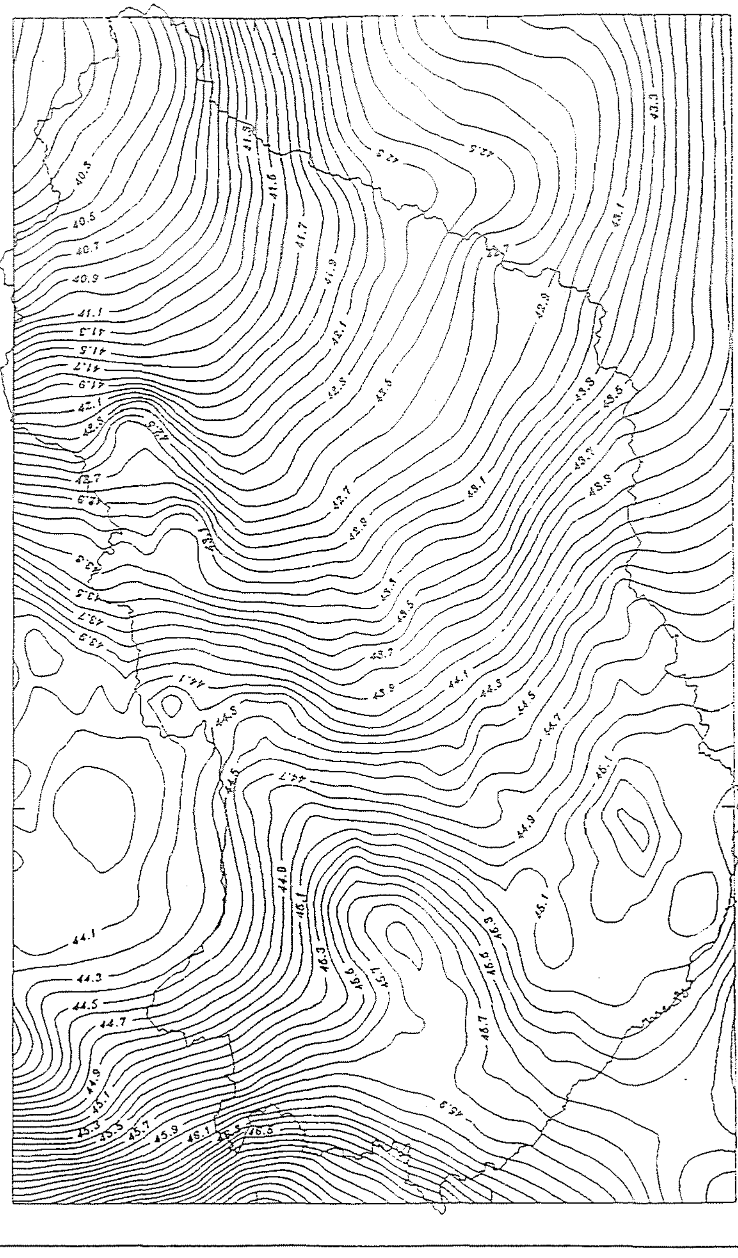

Fig. 6. The HGQ91C gravimetric quasi-geoid solution ($\psi_0 = 1.0°$, OSU89B $n, m = 360$). Reference ellipsoid: geocentric GRS84. Contour interval 0.1 m

The HGQ91C solution is presented in *Fig. 6.* For the computations, the flat approximations of the Stokes and Vening Meinesz integrals were used. The height anomalies were determined for a full rectangle covering the entire territory of Hungary with grid knots over it. The dimensions of the rectangle are 33 × 53 knots with 9600 m mesh size. All the solutions are available in digital form together with an interpolation program which can provide height anomalies at optional points within the computational area.

Quality Analysis

At the moment, five different gravimetric quasi-geoid solutions are available for the territory of Hungary. It would be desirable to decide which the most reliable solution is. There are 2 + 1 different possibilities to compare the results with independent quantities, or simply to make a comparison between the solutions.

Comparison of Solutions

It is easy to derive the difference maps of solutions, but this method is not suitable for real quality analysis. Theoretically, if the input gravity data and the geopotential model would be errorless, the solution differences should be zero. Two of the difference maps are presented here (*Fig. 7, 8*). It is clearly seen that in some areas big differences exist. These maps suggest that the solutions do not cover the entire frequency domain. The proof of this statement requires further investigations.

Comparison of Gravimetric and Astronomic Deflections of the Vertical

By applying the same truncation technique to the Vening Meinesz integral (*Eq.* 4), five different sets of gravimetric deflections were computed with the use of the same parameters as at the height anomaly computations (*Table 2*). The computed deflection components (ξ^G, η^G) were compared with the measured astronomic deflections which were transformed into the GRS80 geocentric system before. One difference set is represented in *Fig. 9.* The statistics (*Table 2*) of the five solutions shows very interesting behaviour. The best result is given by the HGQ91C solution, all the others had worse statistics. The conclusion is that the increase of the truncation cap radius improves the solution but there is a radius limit, and over it the reliability of the solution becomes worse. The reason for it should be the bad quality of the gravity data over the Hungarian border. This re-

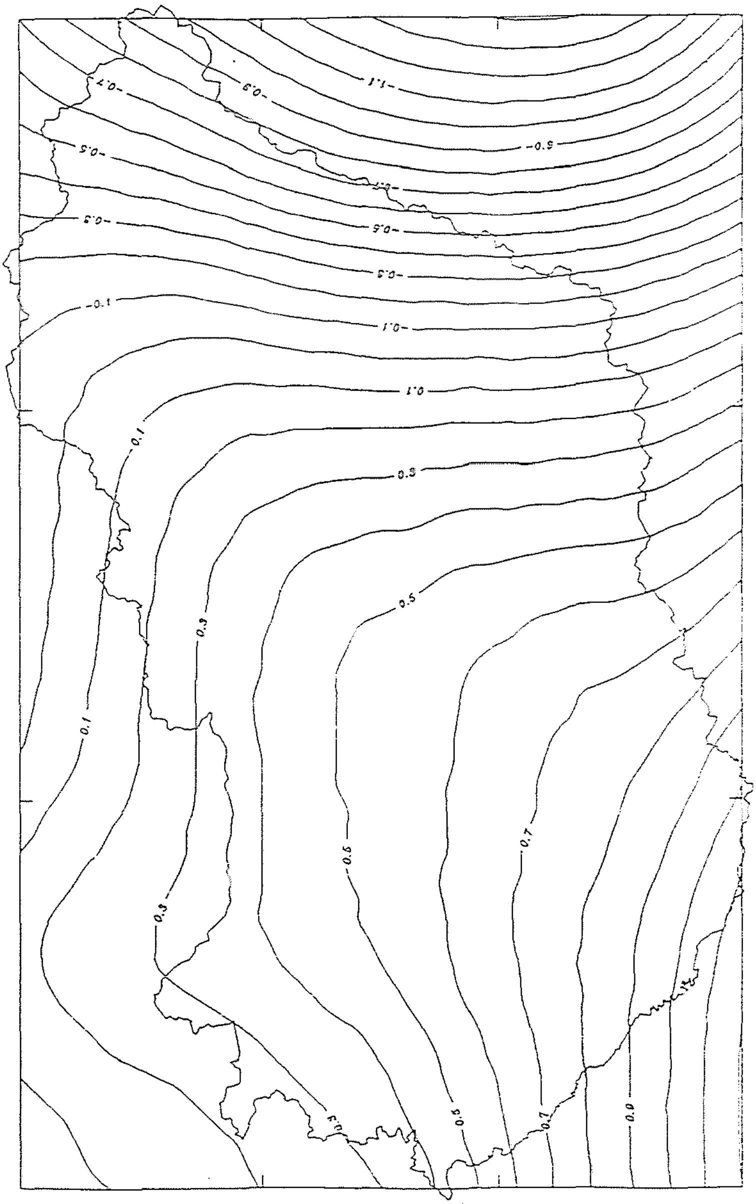

Fig. 7. The difference of HGQ91E and HGQ91C solutions. Contour interval 0.1 m

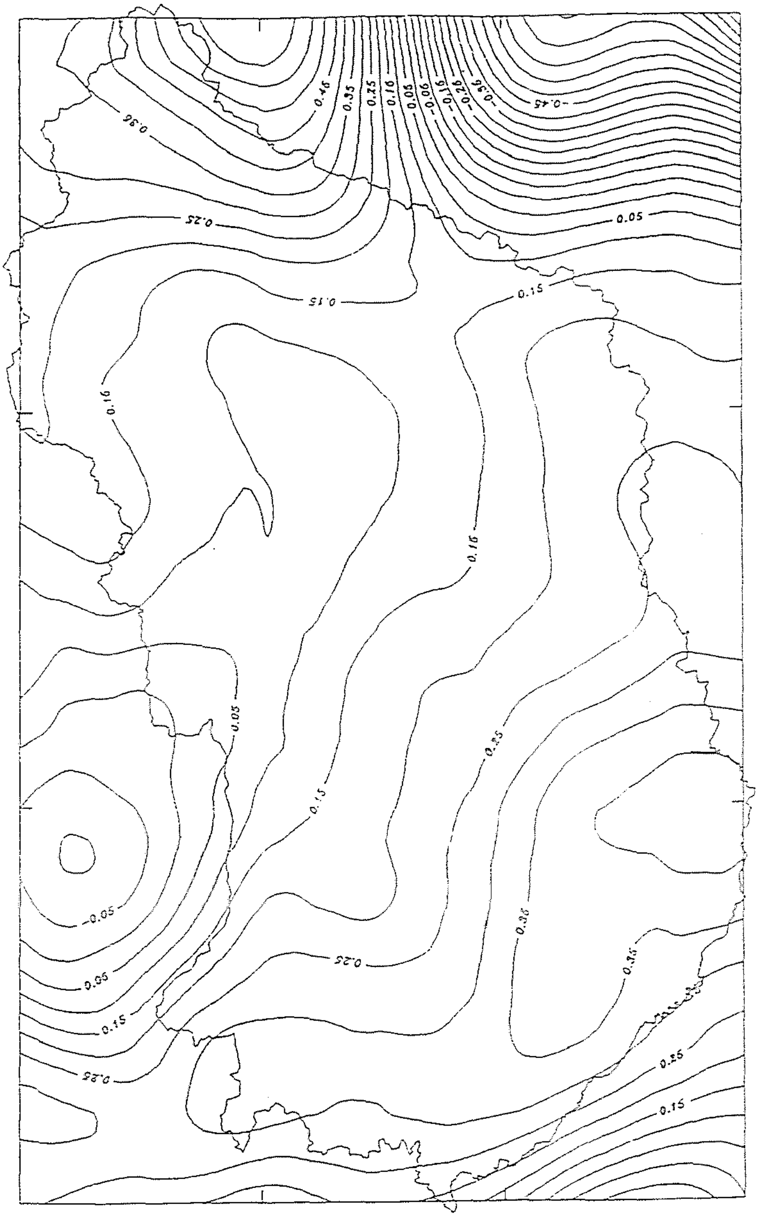

Fig. 8. The difference of HGQ91C and HGQ91A solutions. Contour interval 0.05 m

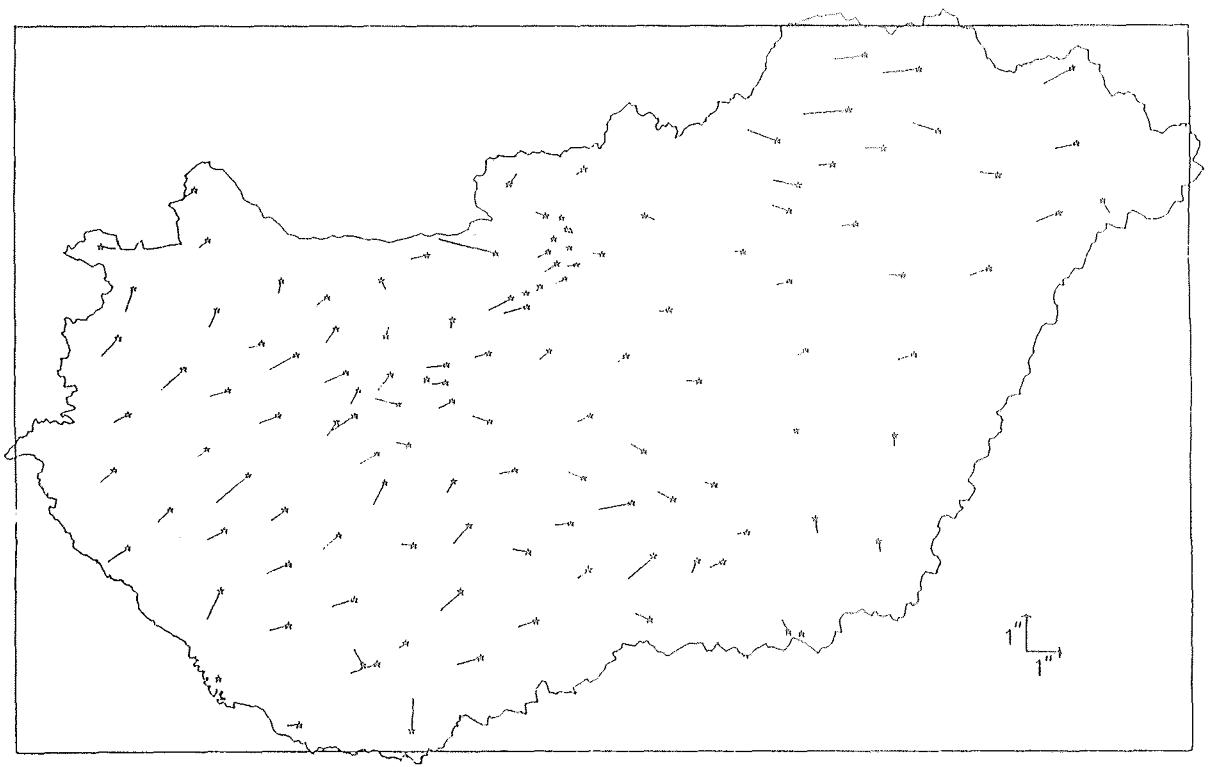

Fig. 9. The differences of the gravimetric (HGQ91C) and astronomic deflections of the vertical

Table 2
The statistics of the gravimetric-astronomic deflection difference solutions in 122 points.

Solution	R.M.S. errors	
	$\xi['']$	$\psi['']$
HGQ91A	1.15	1.49
HGQ91B	1.03	1.41
HGQ91C	0.93	1.45
HGQ91D	0.96	1.51
HGQ91E	1.34	1.32

sult seems to be a good evidence of the fact that the improvement of the gravity data must be considered as the main task of the future research.

Comparison of Gravimetric and GPS/levelling Derived Geoidal Heights

This method should be the primary tool for checking the computed geoid (ENGELIS et al, 1985; DENKER and WENZEL, 1987) but till now, only at few points were available GPS/levelling derived geoidal height differences. So far, only at the north-western part of the country were carried out preliminary comparisons showing about 1–2 ppm relative accuracy of the solutions. In 1991, the so-called zero-order GPS network (20 points covering the whole country) was measured, and the results provided a very good basis for the further comparison study.

Summary and Conclusions

As a result of a three-year research at the SGO, which was partly supported by the HAS OTKA Research Fund, detailed gravimetric quasi-geoid solutions and gravimetric deflections of the vertical are available for the territory of Hungary. These solutions were based on the Meissl–Ostach-type modification of the truncated Stokes and Vening Meinesz integrals. The computations were done by using the GEOMETR program system, which was developed at the SGO by Ambrus Kenyeres. The gravimetric quasi-geoid solutions are available in digital form together with an interpolation program to compute height anomalies at optional points. The relative accuracy of the geoid solutions is estimated to be 1–2 ppm based on preliminary comparisons with GPS/levelling derived geoidal heights. This estimated accuracy is at the international level. The comparison of the computed gravimetric deflections of the vertical with the measured astronomic deflections suggested the conclusion that the quality of the over-border gravity data should be improved.

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