

The Effect of GSI and m_i on the Stability of 3D Twin Tunnel in Limestone

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Abstract

The Generalised Hoek-Brown (GHB) failure criterion is one of the most used criteria to study the behaviour of the rocks; affected parameters of the Hoek-Brown equation are Geological Strength Index (GSI), intact rock constant (m_i), and Disturbance factor (D). GSI is one of the rock classification systems used to evaluate jointed rocks. In light of this equation, this paper studies the stability of unsupported twin tunnels in a weak rock by changing the mentioned parameters (GSI, m_i) to find the relation between the stability and these parameters under different distances between the centres of the tunnels (L). The tunnels have a circular cross-section with a diameter (B), and they have been modelled in three dimensions using Rocscience software package (RS3). The results showed that the stability of the tunnels, which was represented by the strength reduction factor (SRF), increased as a result of increasing L/B or GSI in the studied range; for m_i , the modelling results showed that the SRF value increased while m_i value was increased.

Keywords

GSI, m_i , GHB, stability analysis, RS3, twin tunnel, tunnels distance, soft rock

1 Introduction

The rock mass is a difficult material for both empirical and modelling studies because of its anisotropic nature, which leads to uncertain behavior [1].

The Generalized Hoek-Brown failure criterion is widely accepted to study rock masses and has been used in projects worldwide [2, 3].

The intact rock constant (m_i) is one of the basic input parameters required for using Hoek-Brown failure criterion. The constant m_i is a fundamental parameter required for the Hoek-Brown (HB) failure criterion in estimating the strength of rock materials [4]. The value of m_i depends on many factors, such as grain sizes and mineral compositions [4]. Estimating m_i value is usually done based on a reference's table as a constant value [5]. The value of the m_i for soft rocks is approximately between 2 and 14, according to the suggested table by [6]. The Geological Strength Index (GSI) is widely used for estimating the strength reduction from an intact rock to a rock mass [7], and there are different ways to determine its value [8]; the determination of GSI is not easy and not exact [9].

The International Society for Rock Mechanics (ISRM) raised the definition of soft rock in 1981 based on Uniaxial

Compressive Strength (UCS). A rock with a UCS range of 0.25–25 MPa is classified as extremely weak to weak rock [10]. Another definition of soft rock given by [11] as any rock with UCS less than 20 MPa; [12] presented a range of value of UCS between 0.6–1.25 as very soft rock or hard soil, while [13] defined the rock in the range of 1–5 MPa as very weak rock.

He [10] determined the geological soft rock and the engineering soft rock within the soft rock concept proposed by ISRM. According to that geological soft rock refers to rocks characterized by low stress, large porosity, poor cementation, broken surface and strong weathering-dependence, which basically contain swelling and loose clayey minerals and/or loose, soft, weak layers. Engineering soft rock occurs when the engineering forces cause significant plastic deformation of the rock mass [10]. The calculated soft rock mass, porous limestone, is a geological soft rock according to the classification of He [10].

The stability of singular and twin tunnels has been studied in different kinds of soils [14–18] or rock masses [19–23].

For this study, RS3 has been used as a modelling program, RS3 has been used in previous modelling works as well, like the stability of caverns [24], tunnels [25], piles and piled raft [26], and slope stability [27].

The stability of dual unsupported square sectioned tunnels in rock masses subjected to surcharge loading, obeying Generalized Hoek-Brown (GHB) failure criterion is investigated by Xiao et al. [23]; the safety factor was calculated with changing center to center distance ratio, cover depth ratio, GSI (50, 60, 70, 80, 90, 100) and m_i (5, 10, 15, 20, 25, 30).

Kumar and Rahaman [28] studied the stability of strip footing, slope stability and rectangular unlined tunnel in rock mass with changing the rock mass parameters like GSI and m_i . For tunnel stability, the values of GSI were (10, 30, 50, 70, 90) and $m_i = 15$; furthermore, the cover-to-height ratio of the tunnel was assigned three values: 1, 2, and 4. The researchers used lower bound limit analysis by power cone programming to calculate the stability number of the tunnel.

Twin tunnels in granite rock mass were studied by Singh et al. [29]; these twin tunnels were constructed under cover of 250 m, and the stability calculation was done by ABAQUS v6.12 software; the primary consideration of the modelling was to study the effect of changing of the tunnels' diameter and the spacing factor (S_r); the diameter of the tunnels assigned to the values (2, 5, and 10 m), while the spacing between the tunnels was calculated by multiplying the diameter value by the space factor, the space factor took the values (0.2, 0.4, 0.6 ... 2).

Hallaji Dibavar et al. [30] investigated the effect of spacing of supported Twin Metro Tunnels in Istanbul on the surface settlement by using FLAC^{3D}. The study presents the results of the investigation of twin tunnels' behaviour as passing through homogenous soil in this section. The entire tunnel is located in Trakya Formation; the twin tunnels have an oval shape; the three considered spacing conditions were: various spacing in the direction perpendicular to the tunnel axis, parallel to the tunnel axis, and when the excavation of twin tunnels are performed at the same time.

This research investigates the effect of the change in GSI, m_i and tunnel distance in shallow tunnels cut into soft limestone on the safety factor and displacements.

2 Methodology and modelling

The tunnels were studied in a circular cross-section with 5 m in diameter (B); the twin tunnels are located at the same elevation, with a 5 m cover above their crown.

The distance between tunnels' centres changed between 1.5, 2.0 and 2.5 m. The study focuses on the affection of changing some parameters (GSI, m_i) on the stability of the tunnels. GSI values were in the range of 30–90, and it was increasing by 10; for each value of GSI, a couple of values of m_i were applied, and the m_i values were in the range (4–12), increasing by 2 for each step.

The rock which hosts the tunnels is porous limestone with UCS = 4.11 MPa, which can be sorted as very weak rock according to [13]. The tunnels were studied under surface load which is located above the middle of the tunnels' length in rectangular shape. There are no supporting structures in the tunnels. A cross section of the studied twin tunnels with the dimensions are shown in Fig. 1.

The modelling has been done in 3D using Rocscience finite element analysis program (RS3). The basic parameters of the GSI and m_i which affect Generalized Hoek-Brown's material model were changing during the modelling process. The input parameters are listed in Table 1. Fig. 2 shows the modelled area in 3D and a cross section in the middle of the tunnels' length. The magnitude of the surface load applied in the middle of the tunnels' length was 0.25 MPa.

There are three values for L/B , for each L/B there are 7 values of GSI, and for each GSI value, there are 5 values of m_i . In the end, 105 models have been created and run,

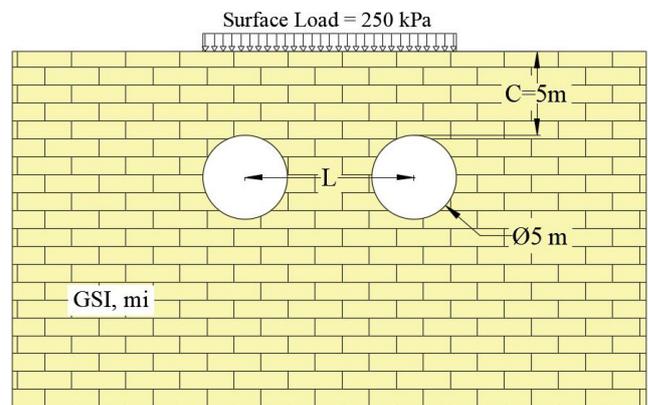


Fig. 1 The cross section of the studied problem, C = cover, L = distance between tunnels' centers

Table 1 The modelling parameters

Parameter	Range
UCS (intact)	4.11 MPa
E (intact)	1.1206 GPa
L/B (distance between tunnels' centres / diameter)	1.5, 2.0, 2.5
GSI	30–90 inc. 10
m_i	4–12 inc. 2
D (disturbance factor)	0

the displacements and the strength reduction factor (SRF) were recorded.

3 Results and discussion

The strength reduction factor, maximum displacements in the entire rock mass and max displacements along the tunnel's

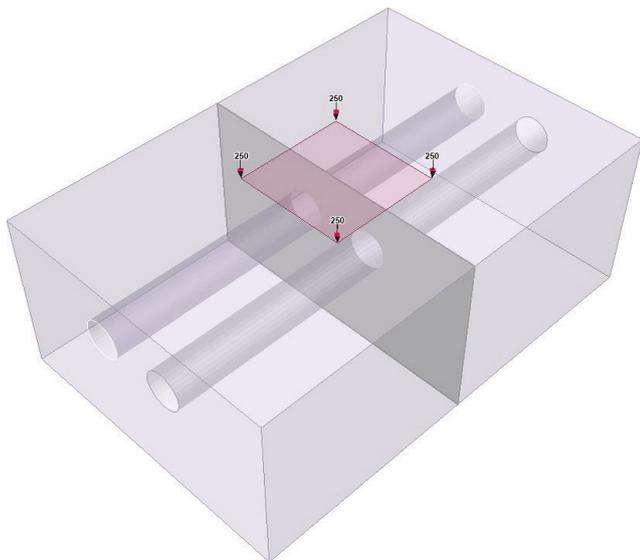


Fig. 2 3D model of the twin tunnel with the surface load 250 kPa, shows the location of the cross section where the displacements of the tunnel's wall recorded

perimeter were recorded for all the models. The maximum displacement points along the tunnels' perimeter are taken from the cross-section in the middle of the tunnels' length, as in Fig. 3, the dark red color refers to maximum displacements while the dark blue refers to zero displacements. The location of the max displacements along the tunnel perimeter is shown with yellow points in Fig. 3. The results of maximum displacements along the tunnels' perimeter and SRF are presented in Tables 2 and 3, respectively, the results are arranged in groups according to GSI, L/B and m_i .

The displacement decreases with increased spacing between the tunnels. When GSI equals or bigger than 60, the displacement of tunnels' perimeter is fixed for each L/B , nevertheless the value of m_i ; when GSI is smaller than 60, the displacements decrease with increasing any of GSI, m_i ,

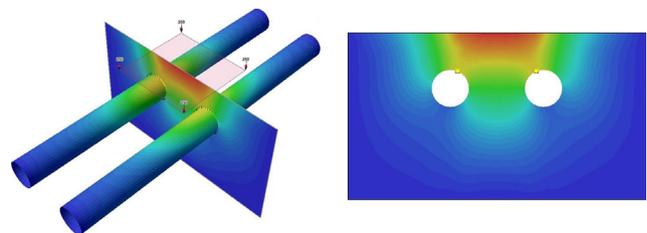


Fig. 3 The cross section at the middle tunnels' length with the locations of recorded displacements points in the tunnels' wall

Table 2 The maximum displacements along the tunnel's perimeter

GSI	m_i	Maximum displacements of tunnels' perimeter			GSI	m_i	Maximum displacements of tunnels' perimeter		
		$L/B = 1.5$	$L/B = 2.0$	$L/B = 2.5$			$L/B = 1.5$	$L/B = 2.0$	$L/B = 2.5$
90	4	3.7	3.2	2.8	50	4	13.4	10.7	9.1
	6	3.7	3.2	2.8		6	12.4	10.5	9.1
	8	3.7	3.2	2.8		8	12	10.4	9.1
	10	3.7	3.2	2.8		10	11.8	10.4	9.1
	12	3.7	3.2	2.8		12	11.8	10.4	9.1
80	4	4	3.5	3.1	40	4	61.8	25.1	20.5
	6	4	3.5	3.1		6	32.6	22.3	19.1
	8	4	3.5	3.1		8	26.6	21.3	18.2
	10	4	3.5	3.1		10	24.7	20.7	17.8
	12	4	3.5	3.1		12	23.8	20.3	17.7
70	4	4.9	4.3	3.7	30	4	68539	12283	62.5
	6	4.9	4.3	3.7		6	12967	60	45.6
	8	4.9	4.3	3.7		8	102.5	49.3	40.1
	10	4.9	4.3	3.7		10	69.8	45.4	38.5
	12	4.9	4.3	3.7		12	57.5	43.4	37.3
60	4	6.9	6	5.3					
	6	6.9	6	5.3					
	8	6.9	6	5.3					
	10	6.9	6	5.3					
	12	6.9	6	5.3					

Table 3 The SRF results of modelling

GSI	m_i	SRF			GSI	m_i	SRF		
		$L/B = 1.5$	$L/B = 2.0$	$L/B = 2.5$			$L/B = 1.5$	$L/B = 2.0$	$L/B = 2.5$
90	4	5.69	6.51	7.34	50	4	1.25	1.45	1.64
	6	5.1	5.8	6.7		6	1.37	1.58	1.77
	8	4.79	5.48	6.24		8	1.46	1.69	1.89
	10	4.61	5.29	6.03		10	1.54	1.76	1.97
	12	4.49	5.15	5.9		12	1.59	1.84	2.04
80	4	3.37	3.9	4.42	40	4	1.02	1.18	1.32
	6	3.22	3.7	4.21		6	1.14	1.29	1.46
	8	3.14	3.64	4.14		8	1.23	1.41	1.56
	10	3.1	3.6	4.1		10	1.29	1.49	1.64
	12	3.1	3.58	4.09		12	1.37	1.57	1.69
70	4	2.23	2.58	2.95	30	4	0.83	0.96	1.07
	6	2.24	2.6	2.96		6	0.95	1.09	1.2
	8	2.26	2.63	2.99		8	1.04	1.19	1.29
	10	2.3	2.67	3.03		10	1.11	1.27	1.35
	12	2.35	2.71	3.08		12	1.17	1.32	1.36
60	4	1.61	1.87	2.13					
	6	1.7	1.96	2.23					
	8	1.78	2.03	2.32					
	10	1.83	2.12	2.39					
	12	1.91	2.19	2.46					

or L/B . Table 2, showed clearly that m_i value has an effect on the displacement when GSI is below 60.

The stability of the twin tunnels increased with the increscent distance between the tunnels in the studied range of L/B , which is similar to the results of [29]. It is also obvious that the SRF value increases with the increase of GSI value. However the effect of the m_i parameter, according to the results, is not similar at different GSI values. Mainly, the increase of the m_i parameter resulted an increase in SRF, but when GSI is around 70, the m_i values have almost no effect on SRF. When GSI is above 70, increasing the m_i value causes a reduction in SRF, according to Fig. 4.

Fig. 5 shows the relationship between GSI and SRF according to the calculation results for different m_i parameters. The three charts are for the three values of L/B . Fig. 5 shows the above-described effect more clearly. The SRF value increases with the increasing m_i parameter in the first part of the diagram. When GSI is between 70 and 80, the effect of the m_i parameter can be negligible, and the previous trend is changed to the opposite. So, when the GSI is bigger than 80, the increase of the m_i parameter causes a decrease in SRF results. Studying the intersection of the curves of $m_i = 4$ and $m_i = 12$, the intersection values for $L/B = 2.5, 2.0,$ and 1.5 are 74.04, 74.02, and 74.33, respectively.

However, the increase of GSI leads to an increase in the stability of the tunnels with different rates according to m_i value.

The affection of GSI can be seen also through the relation between m_i and SRF as in Fig. 6. The grey GSI = 70 line the borderline, which is almost horizontal (the total horizontal line would belong to GSI = 74 value). Above the borderline the lines show decreasing SRF while bellow the borderline it shows increasing SRF value with increasing m_i parameter.

Hoek et al. [13] investigated the relation between GSI and cohesive strength/uniaxial compressive strength with different m_i values. Fig. 7 shows the results from Hoek et al. [13] which looks similar to the graphs of GSI and SRF with varying values of m_i for twin tunnel Fig. 5. On both diagrams, there is a GSI value when using different m_i values in the calculations give the same results, so it depends not on m_i value. Before this point, the higher m_i value resulted higher SRF or cohesive strength, while after this point, the lower m_i value gives higher SRF or cohesive strength. The curves intersect each other; the intersection is at the GSI = 74 value in this study. Studying the diagram of Hoek et al. [13], the intersection point is at the same GSI value if the m_i is between 5 and 14 as in the presented calculation results. The cause of the similarity could be because

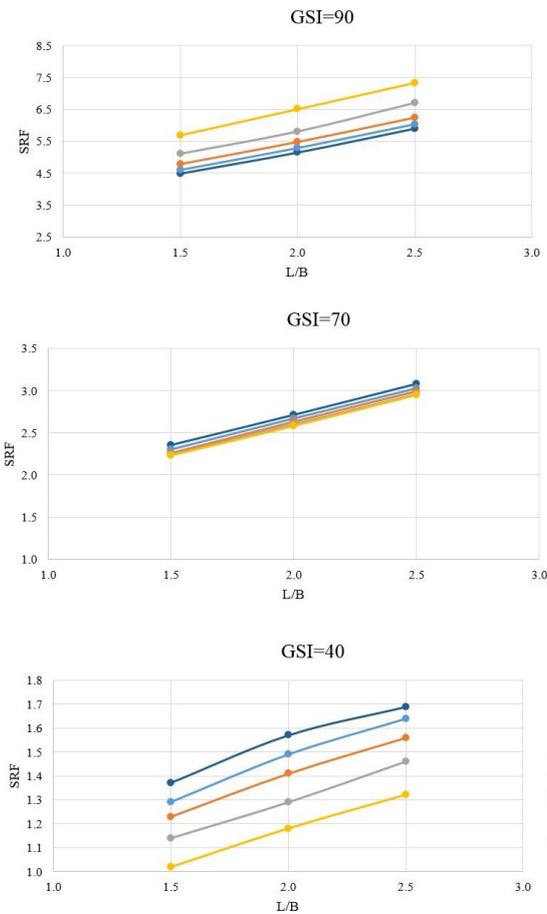


Fig. 4 The relation between SRF and L/B for different values of GSI, (Notice the incompatibility values of the SRF axes)

the SRF and cohesive strength are not independent values. Higher cohesive strength results higher SRF value. The work of Hoek et al. [13] focused on very weak rocks whose compressive strength is in the range of 1–5 MPa [13].

The results of this work showed an increase in SRF as a result of an increase in GSI; for m_i the increasing results showed an increase in SRF before the intersection point and a decrease in SRF after the intersection point as Fig. 5. The work of Xiao et al. [23] showed increasing in stability number as a result of increasing any of GSI or m_i as in Fig. 8. the origin of this difference in both studies may belong to UCS values, which was in the range of weak rock in this work. At the same time, it is not specified in the study of Xiao et al. [23].

The work of Kumar and Rahaman [28] also found that the stability of unlined rectangular tunnels increases continuously as GSI increases.

The work of Hallaji Dibavar et al. [30] showed an increase in Maximum displacements with a decrease in the distance between the twin tunnels when the excavation of twin tunnels is performed at the same time; this result is similar to our results presented in Table 2.

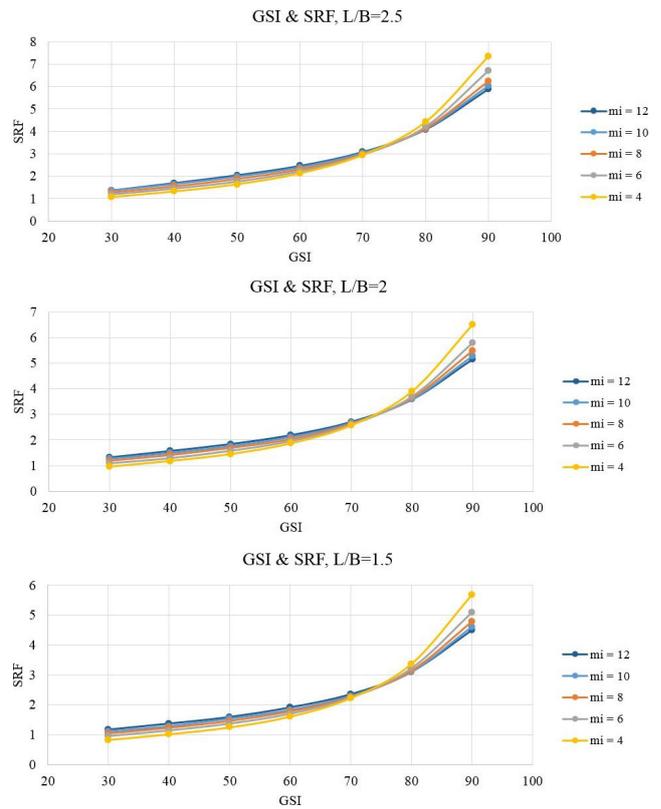


Fig. 5 The relation between SRF and GSI for different values of m_i

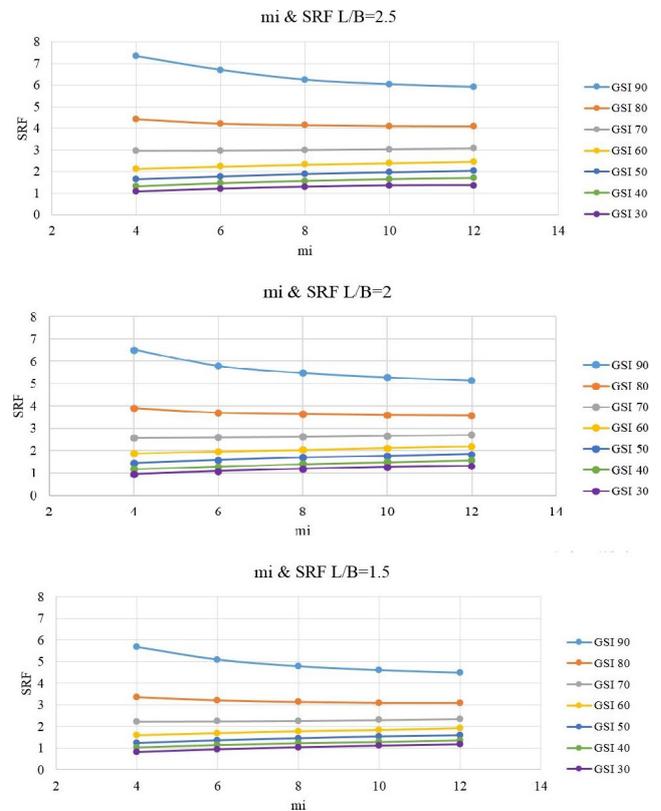


Fig. 6 The relation between SRF and m_i for different values of GSI

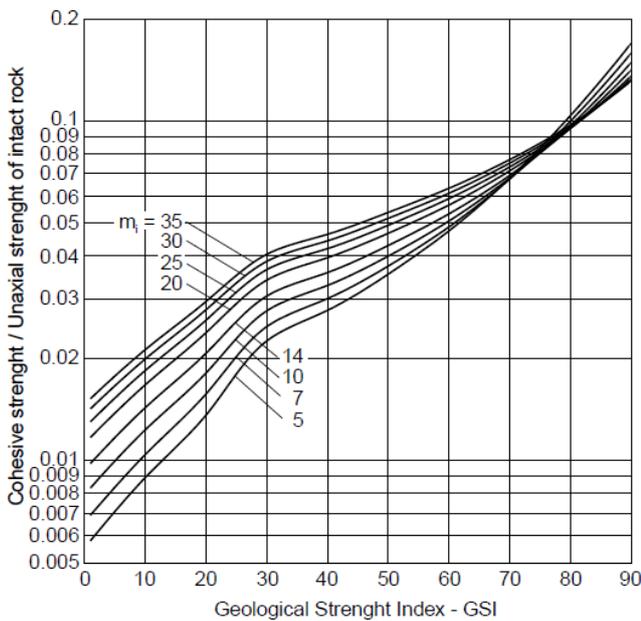


Fig. 7 The relation between GSI and cohesive strength/uniaxial strength of intact rock [13]

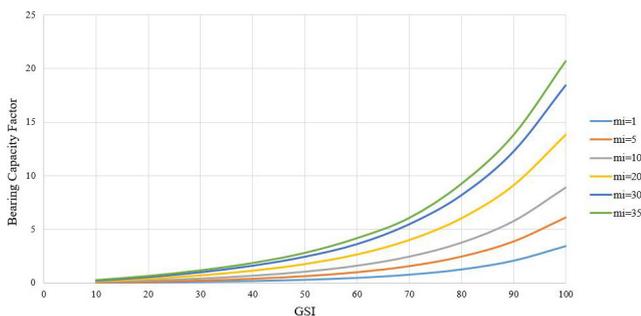


Fig. 8 The relation between GSI and stability number for different values of m_i ; cover/width = 1, center distance/width = 2 (after Xiao et al. [23])

4 Conclusions

The m_i value is an important parameter of the Hoek-Brown failure criterion, which usually slightly affects the stability of twin tunnels, according to the calculations. In spite of this slight effect, it should be considered during the calculation process. In some cases, especially at higher GSI values changing of m_i results wider range of SRF, Fig 6, therefore, the determination of it should be more precise. According to the study of Marinou and Hoek [31] the m_i value can be estimated according to the type of rock, because it is more important the difference between intact rock m_i and rock mass m_b value. But the present study shows that in special cases the m_i value can be important so it should be determined more precisely.

The GSI is an index of rock mass characterization for estimating rock mass strength using the Hoek–Brown criterion, so it is a useful parameter to estimate the strength of the rock mass. Previously, the GSI value was determined

from charts, now it can be calculated according to the parameters of RMR or Q method [32]. The GSI value can easily describe the quality of the rock mass, and its strength can be estimated with the generalized Hoek-Brown failure criterion. Therefore, the stability of a structure, like tunnels, cut into the rock mass depends on the GSI value, as the results presented in Fig. 5. The change in GSI values between 30–70 did not result a big affection on the stability, while the changing in GSI value above 70 resulted a noticeable difference in the stability. The cause of it is that the GSI value has greater effect on the parameters of the generalized Hoek-Brown failure criterion when the GSI value is above 70. It means that the strength of the rock mass calculated by the GSI and generalized Hoek-Brown criterion has considerably increasing with slight changes in the GSI value when $GSI > 70$. This effect usually causes no problems when hard rocks are investigated because the strength of the rock mass of a hard rock is very good when the GSI bigger than 70, so the stability of it is mostly not questionable. Of course, when structural failure is not possible. But this effect can cause problems if a soft rock mass is considered when the GSI value usually above 70 because these rock masses are usually rarely jointed. In this case, the stability can be varied in a wide range, with small changes in the GSI value according to the calculations presented.

Because of this effect, the results can be closer to the reality when the GSI value is not used in these cases. The rare joints can be considered in the model, and the intact rock parameters should be used between the joints.

The value of SRF changed with changing of L/B , the SRF increased by increasing L/B in the studied range (1.5–2.5) Fig. 4.

The changing in SRF value can be divided according to GSI into two ranges; $GSI = 74$ is the dividing value; for each value of GSI smaller than $GSI = 74$, the SRF increases while the m_i value increases. For the GSI values bigger than $GSI = 74$, the SRF increases with the decrease in m_i value. The discussion covers the studied range of m_i : 4–12 (Fig. 5).

The changes in GSI and m_i values both lead to a change in SRF, but the changing in GSI values leads to a more serious change in stability, as shown in Figs. 5 and 6.

The displacements of the tunnels' perimeter are increasing with the decrease in both the GSI value and L/B value in the studied range. In contrast, the m_i values have a negligible effect on the displacements when GSI is low, while it has no impact when GSI is high (Table 2).

For soft rock masses when the $GSI > 74$, it is recommended to use the joints properties in the calculations and

the intact rock parameters between the joints instead of GSI and the generalized Hoek-Brown failure criterion. The abbreviation used in the article listed in Table 4.

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Table 4 Used abbreviations

Abbreviation	Unit	Explanation
GHB	–	Generalised Hoek-Brown
GSI	–	Geological Strength Index
m_i	–	Intact rock constant
D	–	Disturbance factor
L	meter	Distances between the centres of the tunnels
B	meter	Diameter of the circular cross-section of the tunnels
RS3	–	Rocscience software
SRF	–	Strength Reduction Factor

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